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Cooperating in R&D and Advertising

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Abstract

This paper studies the impact of cooperative R&D and advertising on innovation and welfare in a duopolistic industry. The model incorporates two symmetric firms producing differentiated products. Firms invest in R&D and advertising in the presence of R&D spillovers and advertising spillovers. Advertising spillovers may be positive or negative. Four cooperative structures are studied: no cooperation, R&D cooperation, advertising cooperation, R&D and advertising cooperation. R&D spillovers and advertising spillovers always increase innovation and welfare if products are highly differentiated and/or spillovers are sufficiently high. The ranking of cooperation settings in terms of R&D, profits and welfare depends on product differentiation, R&D spillovers and advertising externalities. Firms always prefer cooperation on both dimensions, which is socially beneficial only when advertising and R&D spillovers are sufficiently high.

Key words: R&D, Advertising, Cooperation, Spillovers, Product differentiation, Innovation, Marketing.

JEL Classification: D43; L13; O32

Sommaire

Cette étude analyse l'effet de la coopération technologique et publicitaire sur l'innovation et le bien-être dans une industrie duopolistique. Le modèle incorpore deux firmes symétriques produisant des produits différentiés. Les firmes investissent en R&D et en publicité en présence d'externalités technologiques et publicitaires. Les externalités publicitaires peuvent être positives ou négatives. Quatre scénarios de coopération sont considérés: pas de coopération, coopération technologique, coopération publicitaire et, finalement, coopération technologique et publicitaire. Les externalités technologiques et publicitaires augmentent l'innovation et le bien-être lorsque les produits sont suffisamment différenciés, et/ou lorsque les deux types d'externalités sont suffisamment élevées. Le classement des différents types de coopération en termes de R&D, de profits et de bien-être dépend de la différentiation des produits, des externalités technologiques et des externalités publicitaires. Les firmes préfèrent toujours la coopération technologique et publicitaire, ce qui n'est bénéfique socialement que lorsque les externalités publicitaires et technologiques sont suffisamment élevées.

Mots clés: R&D, Publicité, Coopération, Externalités, Différentiation des produits, Innovation, Marketing.

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1. Introduction[1]

Advertising and innovation are two major instruments for firms to beat the competition, improve future returns effectively and earn extraordinary profits (Askenazy et al. 2016; Hirschey and Weygandt 1985; Drucker 1954). To succeed in the marketplace, corporations require engaging in cooperative R&D and marketing activities (Griffin and Hauser 1996). Firms hire scientists to maintain and develop technology, and hire marketing experts to promote and sell products. If there is no connection between these groups, their ability to synthesize skills, develop and produce successful products decreases which cause firms to suffer. Both marketing and R&D affect the success of new products and they should be analyzed together.

Advertising enables firms to introduce their products to potential consumers, to encourage consumers to purchase the products, to distinguish their products from competitors' and to increase profits. Advertising cooperation between competitors may increase profits. Horizontal advertising cooperation between companies that operate at the same level of the supply chain identifies and develops successful strategies to improve a variety of metrics of performance. Competitors share certain common interests and objectives like the joint development of new products, the evolution of advanced technology and the achievement of high levels of R&D. Cooperation leverages the strengths of each partner to open up new markets or enhance common target markets (Gou et al. 2014).

Additionally, cost reducing R&D increases firms' efficiency and profits. R&D reduces production costs, inducing firms to increase output. Horizontal cooperative R&D yields higher innovation and profits when the information leakage to the rivals (horizontal R&D spillovers) is

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high enough. Cooperative R&D affects innovation and profitability through cost sharing effects (Atallah 2002).

Although R&D is always associated with positive spillovers, which are desirable to the rivals, advertising may generate positive or negative spillovers. Advertising spillovers are positive (negative) if they increase (decrease) the demand for the competitors' products. Sahni (2016) shows that online advertising positively affects competitors' markets. An instance of positive advertising spillovers is the impact of advertising of PepsiCo on the demand of its rival, Coca-Cola. Shapiro (2018) shows that there are positive spillovers from television advertising of prescription psychic-energizers. Advertising has positive externalities on the market size of all rivals if all firms invest in advertising and compete a la Cournot (Cellini and Lambertini 2002).

Friedman (1983) incorporates positive and negative advertising spillovers in an oligopolistic model. He shows that when advertising spillovers are negative, the cost of advertising is increased and profit is reduced. Negative advertising spillovers can materialize by providing specific information, counterfeiting a well-known prestigious brand (sub-brand), and recalling products. For example, advertising a product as Genetically modified organisms (GMO) free brings new information to light about health risks associated with GMO products. This advertising not only decreases the demand for the competitors' products, but it also influences the demand for other products (imperfect substitutes or independent products), which may contain GMO. Another example of negative advertising spillovers is promoting counterfeit products as genuine ones which negatively affects the market of the premium brand. Premium brands attract consumers with their associated exclusive prestige. However, counterfeiting a premium brand enables a variety of consumers to take advantage of the brand name. Thus, the search for exclusivity of the consumers of genuine items induces them to abandon the premium brand (Commuri 2009). Although marketers create a bridge between relevant brands in their brand portfolios to increase marketing efficiency through positive spillovers of brand equity, negative spillovers may occur and affect relative brands that are not directly involved (Lie et al. 2008). For example, prompting a recall of Nescafé products due to a severe quality problem encourages consumers to update their evaluations not just for Nescafé but also for other Nestlé's sub-brands.

Recent studies show that the strategic interaction between advertising and R&D depends on the returns associated with these activities (Kaiser 2005). Advertising and R&D can be strategic substitutes; firms may substitute advertising for R&D when the returns associated with advertising are higher than the returns on R&D. However, advertising and R&D can also be strategic complements. The literature has taken the view that cooperative R&D and advertising have positive effects on firm value.

Previous theoretical studies focused on the effects of cooperative advertising and cooperative R&D on firm value. There are many analyses investigating the effects of R&D spillovers and cooperative R&D strategies on innovation. However, there is no analysis addressing the impact of cooperative R&D and advertising on innovation and welfare. Investigating these effects is essential and helps the regulator to introduce required policies to protect innovation and welfare, notably where horizontal mergers are prohibited. This paper makes several contributions to the literature. First, it is the first paper to model the simultaneous presence of R&D and advertising spillovers, by allowing firms to invest in cost reducing R&D and advertising simultaneously. Advertising spillovers may be positive or negative. Second, the paper studies the effect of cooperative R&D and cooperative advertising by allowing firms to choose R&D and advertising to maximize joint profits. Finally, this paper compares the innovation outcomes associated with different cooperation settings and classifies them in terms of R&D, profits and welfare.

In a two-stage game, two symmetric duopolists producing differentiated products invest in R&D and advertising in the first stage and then compete in output in the second stage. R&D decreases the production cost of the investing firm, and the competitor's cost through R&D spillovers. Advertising increases own demand and increases (reduces) the rival's demand through positive (negative) advertising spillovers.

It is found that higher R&D and advertising spillovers always increase innovation, output and welfare when goods are independent. When goods are imperfect substitutes or homogeneous, the impact of R&D and advertising spillovers on innovation, output, and welfare depend on advertising spillovers, R&D spillovers, product differentiation and cooperative structures. The study of the ranking of cooperation settings shows that cooperative advertising always reduces innovation when advertising spillovers are negative. Indeed, internalizing positive externalities always promotes innovation and thereby, the type of cooperation which internalizes larger positive externalities yields higher innovation. The study of the strategic interaction between competitors' R&D and advertising shows that the advertising of a firm complements its competitor's R&D and advertising when advertising spillovers are positive and large enough. The R&D of a firm complements its competitor's R&D and advertising if R&D spillovers are high enough (with respect to product differentiation). Firms always prefer cooperation on both dimensions, which is socially beneficial only when advertising and R&D spillovers are sufficiently high; thus there is a serious scope for market failure.

The paper is organized as follows. A brief literature review of empirical studies is provided in section 2. The model is presented in section 3. Section 4 examines the strategic interaction between competitors' advertising and R&D. The impact of spillovers[2] on innovation, output and welfare is studied in section 5. The four scenarios (no cooperation, R&D cooperation, advertising cooperation, R&D and advertising cooperation) are compared in section 6. Section 7 provides a brief summary and conclusion.

2. Literature Review

The majority of empirical analyses study the cost sharing effects of advertising and R&D cooperation and find a positive impact of cooperative advertising and R&D on profits (Gasmi et al. 1992; Vanhaverbeke et al. 2002). Sridhar et al. (2014) study U.S. high technology manufacturing firms. Focusing on the long-term effects, they find that advertising and inventory holdings increase sales, whereas R&D does not. Advertising and R&D increase firm value, whereas inventory holding does not.

Most of the R&D intensive industries, like the pharmaceutical and automotive industries have large advertising expenditures (Matraves 1999; Kwong and Norton 2007; Kaiser 2005). Focusing on the competitive roles of advertising and R&D, evidence shows that advertising and R&D are critical to price competition and the formation of market structure in the global pharmaceutical industry (Matraves 1999).

A large literature in economics and management presents various motives that encourage firms to collaborate in R&D (Nielsen 2002; Nooteboom 1999). A number of empirical studies explore the determinants of R&D cooperation (Kleinknecht and Reijnen 1992; Fritsch and Lukas 2001; Tether 2002; Belderbos et al. 2004). A major finding of recent contributions is that the goals and determinants of R&D cooperation differ depending on the type of R&D and

[2] Spillovers refer to R&D and advertising spillovers.

cooperation partner. Fritsch and Lukas (2001) study a sample of German manufacturing firms and find that process innovations are associated with cooperation with suppliers, whereas product innovations are associated with customer cooperation. Tether (2002) studies a sample of innovating firms from the UK; he finds that R&D cooperation is followed by firms pursuing disruptive rather than incremental innovations.

Siebert (1996) studies the effect of R&D alliances on firm performance using U.S. joint ventures. He shows that the effect of R&D on the profit margin is larger for cooperating than for non-cooperating firms. Aschhoff and Schmidt (2008), from the study of firm level data from the German Innovation Survey, find that cooperative R&D strengthens cost reductions and has a positive impact on firm success with market novelties. The positive effects of cooperative R&D are reinforced if firms cooperate in the introduction and marketing of innovative (new) products.

Kwoka (1993) studies the U.S. auto industry and finds that advertising positively affects a car's model sales; however, the effect is temporary and limited. Seldon and Doroodian (1989) find that although advertising positively affects the demand for cigarettes in the U.S., health warnings mitigate this effect. They show that the industry responds to the health warnings by increasing advertising. The results confirm the effects of negative advertising spillovers of the health warnings on the cigarette industry. Nerlove and Waugh (1961) find a positive impact of advertising in the U.S. orange industry.

Despite the importance of advertising alliances, few empirical studies address the effectiveness of alternative strategies for cooperative advertising. Gasmi et al. (1992) study potential marketing coordination in the Cola market (Nash behaviour, Stackelberg leadership, collusion) using data for two decades. They find evidence which confirms joint marketing alliances. Wang et al. (2004) find a similar conclusion for the U.S. butter and margarine industry.

Kadiyali (1996) studies the U.S. photographic film industry where evidence shows cooperation on both advertising and price. Roberts and Samuelson (1988) address the joint profit maximizing choice of advertising in the U.S. cigarette market.

Cooperative advertising enhances target markets when advertising spillovers are positive and/or advertising is not comparative. Anderson et al. (2016), from the study of U.S. TV advertising regarding over-the-counter analgesics, find that there are negative effects of comparative advertising on competitors. Engaging in comparative advertising negatively affects the rival's market and reduces joint profits where firms cooperate in advertising. Comparative advertising causes more damage to the targeted competitor than benefit to the advertiser. The analysis confirms the results of this paper that cooperative advertising reduces advertising when advertising spillovers are negative.

Matraves (1999) and Kwong and Norton (2007) find that industry characteristics, R&D intensity, returns on R&D and marketing determine how the strategic interaction of advertising and R&D between firms may vary. Advertising and R&D may be either strategic substitutes or strategic complements. If the returns associated with advertising are higher than returns on R&D, firms may substitute advertising for R&D. This mechanism is more significant when firms face credit constraints or have to pursue short-run objectives. However, evidence shows that advertising and R&D can also be complements, as with pharmaceuticals who have high advertising expenditures and large R&D budgets. Askenazy et al. (2016) study the French manufacturing industry. They employ a model including both static and dynamic interactions between R&D, advertising, and the competitive environment. They find that for a given competitive environment, quality leaders invest more in advertising to extract more rents.

Additionally, the inverted-U relation between competition and R&D remains with the introduction of advertising. Indeed, more competition is associated with more advertising.

Although there is a large theoretical and empirical literature analyzing the importance of R&D and marketing along with the effects of cooperative R&D and advertising on firm value, performance, and strategies, there is no theoretical or empirical literature studying the impact of R&D and advertising cooperation on innovation, profits, and welfare. This paper contributes to the literature by its unique approach, and also by modelling explicitly the interactions between innovation and advertising.

3. The Model

There are two symmetric firms producing differentiated products and competing a la Cournot. The model follows a two-stage game. In the first stage, firms choose R&D (x) and advertising (a). In the second stage, they choose output (y). Good *i* is produced by firm *i*, where *i*=1,2. In this model, advertising increases the market demand for the good, has some feature of complementary[3] advertising (Bagwell, 2007), and may or may not provide information. Advertising in this paper does not provide any solid and specific information on price or product characteristics, it also does not affect product differentiation. This model shares some features with the model by Becker and Murphy (1993). A consumer consumes two goods, 1 and 2. Consumers have a stable set of preferences and advertising is complementary to consumption.

The consumer's utility function is given by

$$U(y_0, y_1, y_2) = k(y_1 + y_2) - \frac{(y_1^2 + y_2^2)}{2} - by_1y_2 + (a_1 + \theta a_2)y_1 + (\theta a_1 + a_2)y_2 + y_0 \qquad (1)$$

[3] The complementary view admits that advertising may contain information and influence consumer behavior accordingly. However, the consumer may value "social prestige," and advertising by a firm may be an input that contributes toward the prestige that increases satisfaction when the firm's product is consumed.

where y is output, a denotes advertising, $\theta \in [-1,1]$ is a parameter measuring the advertising spillover, $b \in [0,1]$ is a parameter indicating the degree of product substitutability (negatively related to product differentiation). Final goods are homogeneous if $b = 1$, imperfect substitutes if $0 < b < 1$ and independent if $b = 0$. y_0 is the numeraire, given by the budget constraint $y_0 = M - p_1 y_1 - p_2 y_2$, where M is income and p is price. Advertising by a firm generates a positive (negative) spillover (θ) for the value of the other product to the consumer.

Maximizing utility over y_1 and y_2 yields the following demand functions:

$$p_i(y_i, y_j, a_i, a_j) = k - y_i - b\, y_j + a_i + \theta a_j \qquad i,j = 1,2 \qquad i \neq j$$

The inverse demand function shows that the demand of each product depends on the advertising choices of all firms. In this setting, advertising causes a parallel shift in the demand curve, with no slope effect. Advertising improves value (by increasing the vertical demand intercept of the inverse demand curve). The rival's demand is negatively (positively) affected by a firm's advertising when $\theta < 0$ ($\theta > 0$).

Each firm invests in advertising and R&D. The dollar cost of x units of R&D for firm i is $\gamma\, x_i^2/2$, where x_i represents the R&D output of firm i, and $\gamma > 0$ is a cost parameter. The dollar cost of a units of advertising for firm i is $\lambda\, a_i^2/2$, where a_i represents advertising by firm i, and $\lambda > 0$ is a cost parameter. Total R&D output is denoted by X and total advertising is denoted by a:

$$X = x_1 + x_2$$

$$a = a_1 + a_2$$

Each unit of R&D conducted by a firm reduces its cost by one dollar, and reduces the competitor's cost by β dollars, $\beta \in [0,1]$. The magnitude of β depends on several factors such as absorptive capacities, technological trajectories, efficiencies of communication channels, etc. Undertaking no R&D, firms have a constant unit production cost of α. The unit cost for firm *i* is:

$$c_i = \alpha - x_i - \beta x_j \qquad i,j = 1,2 \qquad i \neq j$$

The cost function shows that the final cost of each firm depends on the R&D choices of all firms. The game has two stages. In the first stage, each firm chooses its R&D and advertising simultaneously. In the second stage, firms compete a la Cournot.

3.1 Output stage

In the second stage firms choose output to maximize profit non-cooperatively. A firm's problem is:

$$max_{y_i}\ \pi_i = \left(p_i\left(y_i, y_j, a_i, a_j\right) - c_i\right)y_i - \gamma \frac{x_i^2}{2} - \lambda \frac{a_i^2}{2} \qquad i,j = 1,2 \qquad i \neq j$$

Given that firms are symmetric, they follow a symmetric behavior. Maximizing and solving the two f.o.c. simultaneously yields:

$$y_i = \frac{2\left(k-\alpha+a_i+\theta a_j+x_i+\beta x_j\right)-b\left(k-\alpha+\theta a_i+a_j+\beta x_i+x_j\right)}{4-b^2} \qquad i,j = 1,2 \qquad i \neq j$$

$$p_i = \frac{(2-b)k+2(\alpha+a_i-x_i)+2\theta a_j-2\beta x_j+b\left(\alpha-\theta a_i-a_j-\beta x_i-x_j\right)-b^2\left(\alpha-x_i-\beta x_j\right)}{4-b^2} \qquad i,j = 1,2 \qquad i \neq j$$

3.2 Advertising and R&D Stage

In the first stage, firms choose R&D and advertising simultaneously. One noticeable feature of cooperative R&D and advertising is the great variety of possible linkages between firms. Firms may engage in R&D cooperation or in advertising cooperation (or both). To capture the variety of cooperative structures, four scenarios for firm behavior in the first stage are considered. The first scenario is non-cooperative R&D and advertising (*NN*) so that each firm chooses R&D and advertising to maximize profit. The second scenario is cooperative R&D (*RN*) where firms choose R&D cooperatively, but each firm chooses advertising non-cooperatively. The third scenario is cooperative advertising (*NA*), in which firms choose advertising cooperatively, but each firm chooses its R&D non-cooperatively. The fourth scenario is cooperative R&D and advertising (*RA*), where firms determine R&D and advertising cooperatively. Define the set of R&D and advertising choices of all firms as $\varphi \equiv \{x_1, x_2, a_1, a_2\}$. Therefore, the profit of each firm can be rewritten as:

$$\pi_i(\varphi) = (p_i(\varphi) - c_i(\varphi))y_i(\varphi) - \gamma \frac{x_i^2}{2} - \lambda \frac{a_i^2}{2} \qquad i=1,2$$

Welfare is also a function of R&D and advertising, defined as:

$$W(\varphi) = (\pi_1(\varphi) + \pi_2(\varphi)) + CS^{4}$$

In the first stage, under *NN* each firm determines R&D and advertising to maximize profit. Each firm solves the following problem:

$$max_{x_i,a_i}\ \pi_i(\varphi) \qquad i=1,2 \qquad (2)$$

Simultaneously solving the four first order conditions of Eq. (2) gives research and advertising under *NN*:

$$x_i^{NN} = \frac{2\lambda(k-\alpha)(2-b\beta)}{2\gamma(1+\theta)(2-b\theta)+2\lambda(1+\beta)(2-b\beta)-\gamma\lambda(2-b)(b+2)^2} \qquad i=1,2$$

$$a_i^{NN} = \frac{2\gamma(k-\alpha)(2-b\beta)}{2\gamma(1+\theta)(2-b\theta)+2\lambda(1+\beta)(2-b\beta)-\gamma\lambda(2-b)(b+2)^2} \qquad i=1,2$$

$$\pi_i^{NN} = \frac{\gamma\lambda(k-\alpha)^2[2\gamma(2-b\theta)^2+2\lambda(2-b\beta)^2-\gamma\lambda(4-b^2)^2]}{(2\gamma(1+\theta)(2-b\theta)+2\lambda(1+\beta)(2-b\beta)-\gamma\lambda(2-b)(2+b)^2)^2} \qquad i=1,2$$

$$W^{NN} = \frac{\gamma\lambda(k-\alpha)^2[4\gamma(2-b\theta)^2+4\lambda(2-b\beta)^2-\gamma\lambda(3+b)(4-b^2)^2]}{(2\gamma(1+\theta)(2-b\theta)+2\lambda(1+\beta)(2-b\beta)-\gamma\lambda(2-b)(2+b)^2)^2}$$

[4] Consumer surplus is defined as $CS = U(y_0, y_1, y_2) - (p_1y_1 + p_2y_2 + y_0)$.

Under *RN*, firms choose R&D to maximize joint profits, but each firm chooses advertising to maximize its profit.

$$max_{x_1,x_2}\ \pi_1(\varphi)+\pi_2(\varphi) \tag{3}$$

$$max_{a_i}\ \pi_i(\varphi) \qquad i=1,2 \tag{4}$$

Simultaneously solving the four f.o.c. of Eqs. (3) and (4) gives research and advertising under *RN*:

$$x_i^{RN}=\frac{2\lambda(k-\alpha)(\beta+1)(2-b)}{\lambda(2-b)[\gamma(b+2)^2-2(1+\beta)^2]-2\gamma(1+\theta)(2-b\theta)} \qquad i=1,2$$

$$a_i^{RN}=\frac{2\gamma(k-\alpha)(2-b\theta)}{\lambda(2-b)[\gamma(b+2)^2-2(1+\beta)^2]-2\gamma(1+\theta)(2-b\theta)} \qquad i=1,2$$

$$\pi_i^{RN}=\frac{\gamma\lambda(k-\alpha)^2[\lambda(2-b)^2[\gamma(2+b)^2-2(1+\beta)^2]-2\gamma(2-b\theta)^2]}{(2\gamma(1+\theta)(2-b\theta)+\lambda(2-b)[2(1+\beta)^2-\gamma(2+b)^2])^2} \qquad i=1,2$$

$$W^{RN}=\frac{\gamma\lambda(k-\alpha)^2[\lambda(2-b)^2[\gamma(2+b)^2(3+b)-4(1+\beta)^2]-4\gamma(2-b\theta)^2]}{(2\gamma(1+\theta)(2-b\theta)+\lambda(2-b)[2(1+\beta)^2-\gamma(2+b)^2])^2}$$

Under *NA*, firms choose advertising to maximize joint profits, but each firm chooses R&D to maximize its profit.

$$max_{a_1,a_2}\ \pi_1(\varphi)+\pi_2(\varphi) \tag{5}$$

$$max_{x_i}\ \pi_i(\varphi) \qquad i=1,2 \tag{6}$$

Simultaneously solving the four f.o.c. of Eqs. (5) and (6) gives research and advertising under *NA*:

$$x_i^{NA}=\frac{2\lambda(k-\alpha)(2-b\beta)}{\gamma(2-b)[\lambda(b+2)^2-2(1+\theta)^2]-2\lambda(1+\beta)(2-b\beta)} \qquad i=1,2$$

$$a_i^{NA}=\frac{2\gamma(k-\alpha)(2-b)(1+\theta)}{\gamma(2-b)[\lambda(b+2)^2-2(1+\theta)^2]-2\lambda(1+\beta)(2-b\beta)} \qquad i=1,2$$

$$\pi_i^{NA}=\frac{\gamma\lambda(k-\alpha)^2[\gamma\lambda\left(4-b^2\right)^2-2\lambda(2-b\beta)^2-2\gamma(2-b)^2(1+\theta)^2]}{(2\lambda(1+\beta)(2-b\beta)+\gamma(2-b)[2(1+\theta)^2-\lambda(2+b)^2])^2} \qquad i=1,2$$

$$W^{NA}=\frac{(\gamma\lambda(k-\alpha)^2\left[(2-b)^2\gamma\left[\lambda(2+b)^2(3+b)-4(1+\theta)^2\right]-4\lambda(2-b\beta)^2\right)]}{(2\lambda(1+\beta)(2-b\beta)+\gamma(2-b)[2(1+\theta)^2-\lambda(2+b)^2])^2}$$

Finally, under *RA*, firms determine R&D and advertising cooperatively to maximize joint profits.

$$max_{x_1,x_2,a_1,a_2}\ \pi_1(\varphi)+\pi_2(\varphi) \tag{7}$$

Simultaneously solving the four f.o.c. of Eq. (7) gives research and advertising under *RA*:

$$x_i^{RA}=\frac{2\lambda(k-\alpha)(\beta+1)}{2\gamma(1+\theta)^2+2\lambda(1+\beta)^2-\lambda\gamma(b+2)^2} \qquad i=1,2$$

$$a_i^{RA}=\frac{2\gamma(k-\alpha)(\theta+1)}{2\gamma(1+\theta)^2+2\lambda(1+\beta)^2-\lambda\gamma(b+2)^2} \qquad i=1,2$$

$$\pi_i^{RA}=\frac{(k-\alpha)^2\gamma\lambda}{\gamma\lambda(2+b)^2-2\gamma(1+\theta)^2-2\lambda(1+\beta)^2} \qquad i=1,2$$

$$W^{RA}=\frac{\gamma\lambda(k-\alpha)^2[\gamma\lambda(2+b)^2(3+b)-4\gamma(1+\theta)^2-4\lambda(1+\beta)^2]}{(2\gamma(1+\theta)^2+2\lambda(1+\beta)^2-\gamma\lambda(2+b)^2)^2}$$

4. Strategic Interaction of R&D and Advertising

This section studies when R&D and advertising are strategic complements or strategic substitutes. This analysis helps us understand the marginal gains and losses from investments in R&D and advertising.

Proposition 1.

(i) x_j *and* x_i *are strategic complements when R&D spillovers are high enough and/or product substitutability is low enough; otherwise, they are strategic substitutes.*

(ii) x_j *and* a_i *are strategic complements when R&D spillovers are high enough and/or product substitutability is low enough; otherwise, they are strategic substitutes.*

(iii) a_j *and* x_i *are strategic complements when advertising spillovers are high enough and/or product substitutability is low enough; otherwise, they are strategic substitutes.*

(iv) a_j *and* a_i *are strategic complements when advertising spillovers are high enough and/or product substitutability is low enough; otherwise, they are strategic substitutes*

Table 1 shows a summary of the results.[5]

Table 1. Strategic interaction

	$b = 0$	$b > 0$
$\partial^2\pi_i/\partial x_i\partial x_j$	$+$	$+ if \beta > b/2$ $- if \beta < b/2$
$\partial^2\pi_i/\partial a_i\partial x_j$	$+$	$+ if \beta > b/2$ $- if \beta < b/2$
$\partial^2\pi_i/\partial x_i\partial a_j$	$+ if\ \theta > 0$ $- if\ \theta < 0$	$+ if \theta > b/2$ $- if \theta < \pm b/2$
$\partial^2\pi_i/\partial a_i\partial a_j$	$+ if\ \theta > 0$ $- if\ \theta < 0$	$+ if \theta > b/2$ $- if \theta < \pm b/2$

Part *i* of Proposition 1 states that when β is high enough and products are highly differentiated, an increase in R&D by a firm always increases the marginal profit of R&D to its rival. Higher R&D of a firm reduces its production cost and the rival's through β, benefits both, inducing them to increase output and R&D further. Thus, R&D expenditures are strategic complements when β is high enough and/or b is low. Higher product substitutability increases competition between firms, increasing firms' concerns about the cost reduction effects of their R&D on the rival. Thus, firms increase R&D when the benefit of incoming technological information dominates the negative effects of outgoing knowledge spillovers. The gain is

[5] Detailed analysis and proofs are provided in the Appendix.

significant when β is high enough and b is low. When β is low and/or b is sufficiently high, an increase in R&D by a firm reduces the net benefit of R&D to the rival due to the intense competition, inducing the rival to reduce R&D. In this case, R&D activities are strategic substitutes.

Part *ii* of Proposition 1 states that an increase in R&D by a firm always increases the marginal profit of advertising to the rival when β is high enough and products are highly differentiated. A firm benefits from the rival's R&D through β. When products are highly differentiated, firms do not worry about the cost reduction effects of their R&D on the rival. Thus, the firm increases R&D. This benefits the rival, reduces its production costs, and induces it to increase output. Indeed, higher output increases the net benefit of advertising and induces the rival to increase advertising. When goods are substitutes, competition is intense which requires firms' carefulness about the effects of their activities on the rival. Thus, x_j and a_i are strategic substitutes when β is low and/or b is high.

Part *iii* of Proposition 1 states that an increase in advertising by a firm increases the marginal benefit of R&D to the rival when θ is high enough and products are highly differentiated. Higher advertising increases the demand of the competitor's products through θ, and induces it to increase output. Higher output increases the net benefit of R&D and induces the competitor to increase its R&D. Thus, a_j and x_i are strategic complements when θ is high enough and/or b is low enough. Moreover, when θ is sufficiently low (negative) and/or b is sufficiently high, a firm suffers from higher advertising by the competitor, reducing its output and R&D. Thus, in this case, a_j and x_i are strategic substitutes.

Part *iv* of Proposition 1 states that an increase in advertising by a firm increases the net benefit of advertising to the rival when goods are highly differentiated and θ is high enough. In this context, firms benefit from one another's advertising due to its positive effect on demand. Indeed, higher advertising of a firm reduces the demand of the competitor if θ is negative. When products are substitutes, firms are worried about the effects of their advertising on the rival's demand. Thus, firms increase advertising when b is low and θ is high enough such that the gains from incoming advertising spillovers more than offset the negative effects of outgoing spillovers on demand. Otherwise, firms reduce advertising.

We see that high R&D spillovers, high advertising spillovers, and low product substitutability contribute to strategic complementarity between actions. Whereas low R&D spillovers, low advertising spillovers, and high product substitutability contribute to strategic substitutability between actions.

5. The Impact of Spillovers on R&D and Welfare

This section addresses the effects of R&D and advertising spillovers on innovation, output, and welfare where products are differentiated and cooperative structures are varied.

Proposition 2. *(i) Under NN and NA, an increase in R&D spillovers increases innovation, output, and welfare if they are low enough. (ii) Under RN and RA, an increase in R&D spillovers always increases innovation, output, and welfare.*

Table 2 summarizes the results of Proposition 2.

Table 2. Summary of comparative statics

	No Cooperation	R&D Cooperation	Advertising cooperation	R&D and Advertising Cooperation
$\partial X/\partial\beta$	$+if\ b < 2/(1+2\beta)$ $-\ if\ b > 2/(1+2\beta)$	$+$	$+if\ b < 2/(1+2\beta)$ $-if\ b > 2/(1+2\beta)$	$+$
$\partial a/\partial\beta$	$+if\ b < 2/(1+2\beta)$ $-if\ b > 2/(1+2\beta)$	$+$	$+if\ b < 2/(1+2\beta)$ $-if\ b > 2/(1+2\beta)$	$+$
$\partial Y/\partial\beta$	$+if\ b < 2/(1+2\beta)$ $-if\ b > 2/(1+2\beta)$	$+$	$+if\ b < 2/(1+2\beta)$ $-if\ b > 2/(1+2\beta)$	$+$
$\partial\pi_i/\partial\beta$	$+\ if\ \theta \leq b/2\ \&\ \beta \leq b/2$ $-\ if\ \theta > b/2\ \&\ \beta > b/2$	$+\ if\ \theta \leq b/2$ $\pm\ if\ \theta > b/2$	$+\ if\ \beta \leq b/2$ $\pm\ if\ \beta > b/2$	$+$
$\partial W/\partial\beta$	$+if\ b < 2/(1+2\beta)$ $-\ if\ b > 2/(1+2\beta)$	$+$	$+if\ b < 2/(1+2\beta)$ $-\ if\ b > 2/(1+2\beta)$	$+$

An increase in β reduces the rival's production costs and is not desirable. Under *NN* and *NA*, higher β increases innovation when the benefit from incoming technological knowledge dominates the negative effects of outgoing information. In this context, higher β increases the net benefit of R&D, yielding higher innovation and output. Higher output increases the net benefit of advertising and induces firms to increase advertising. Higher advertising increases demand and thereby yields higher profit. Higher output also increases consumer satisfaction and welfare. As product substitutability increases, competition becomes intense which requires firms' carefulness about the effect of their innovation on the rival. Indeed, firms reduce innovation when the sum of the effects of β and b on profit is negative.

Under *RN* and *RA*, firms choose R&D cooperatively to maximize joint profits. In this case, firms do not worry about the leakage of the technical information to the competitor. Thus, higher β increases innovation. Higher innovation reduces production costs and induces firms to increase output. Higher b increases competition, imposes some pressure on the demand, and induces firms to reduce output which is more than offset by higher β, and thereby firms increase output. Higher output increases the net benefit of advertising and induces firms to increase advertising. Consumers benefit from higher output and welfare increases as a result.

Under *RN*, an increase in β increases profit if θ is low enough ($\theta \leq b/2$) such that the benefit from incoming advertising spillovers dominates the negative effect of outgoing advertising spillovers. *RN* does not internalize θ, thus under *RN* competitors suffer from θ when it is high, whereas they benefit from θ under *RA*, where they choose R&D and advertising cooperatively.

Proposition 3. *(i) Under NN and RN, an increase in advertising spillovers increases (decreases) innovation, output, and welfare when advertising spillovers are low (high) enough; (ii) Under NA and RA, an increase in advertising spillovers always increases innovation, output and welfare.*

Table 3 summarizes the results of Proposition 3.

Table 3. Summary of comparative statics

	No Cooperation	R&D Cooperation	Advertising cooperation	R&D and Advertising Cooperation
$\partial X/\partial\theta$	$+if\ b < 2/(1+2\theta)$ $-if\ b > 2/(1+2\theta)$	$+if\ b < 2/(1+2\theta)$ $-if\ b > 2/(1+2\theta)$	+	+
$\partial a/\partial\theta$	$+if\ b < 2/(1+2\theta)$ $-\ if\ b > 2/(1+2\theta)$	$+if\ b < 2/(1+2\theta)$ $-\ if\ b > 2/(1+2\theta)$	+	+
$\partial Y/\partial\theta$	$+if\ b < 2/(1+2\theta)$ $-\ if\ b > 2/(1+2\theta)$	$+if\ b < 2/(1+2\theta)$ $-\ if\ b > 2/(1+2\theta)$	+	+
$\partial\pi_i/\partial\theta$	$+\ if\ \theta \le b/2\ \&\ \beta \le b/2$ $-\ if\ \theta > b/2\ \&\ \beta > b/2$	$+\ if\ \theta \le b/2$ $-if\ \theta > b/2$	$+\ if\ \beta \le b/2$ $-\ if\ \beta > b/2$	+
$\partial W/\partial\theta$	$+if\ b < 2/(1+2\theta)$ $-\ if\ b > 2/(1+2\theta)$	$+if\ b < 2/(1+2\theta)$ $-\ if\ b > 2/(1+2\theta)$	+	+

An increase in θ increases the demand for the competitor's product which is not desirable. *NN* and *RN* do not internalize θ. Higher θ increases profits if the gains from incoming advertising spillovers dominate the negative effects of outgoing advertising spillovers on demand. Indeed, firms may worry about the effect of higher θ on the demand of the competitor's product. This reduces the net benefit of advertising. Indeed, higher *b* boosts competition, induces firms to reduce output, and thereby increases firms' concerns about the impact of θ on the rival's demand. Thus, higher θ benefits firms and induces them to increase output and innovation when products are highly differentiated and θ is sufficiently low. In this context, higher innovation reduces production cost and benefits firms (this effect is significant when firms also cooperate in R&D) and induces firms to increase output further. Higher output increases the net benefit of advertising and induces firms to increase advertising. Conclusively, higher output increases consumer surplus and welfare.

Under *NA* and *RA*, firms choose advertising cooperatively to maximize joint profits. Higher θ increases the rival's demand, benefitting it. This encourages the competitor to increase output and advertising. Higher advertising increases market size, benefits both firms, and induces them to increase output further. Higher output increases the value of R&D and induces firms to increase R&D. Higher R&D benefits both firms and induces them to increase innovation further to benefit from the value of its cost reduction effects (this effect is significant when firms cooperate in R&D). Higher b increases competition, induces firms to reduce output which increases firms' concerns about the effects of θ on demand. However, cooperating in advertising and internalizing the effects of θ mitigate these concerns, offsetting the negative effects of b on output and profits specifically when firms also cooperate in R&D. Ultimately, firms increase output and innovation as a result of higher θ. Consumers benefit from higher output and welfare increases.

Note that under *NA*, where firms choose own R&D non-cooperatively, an increase in θ increases profits if β is low enough ($\beta \leq b/2$); otherwise, it reduces profits.

6. Comparison of Cooperative Structures

In this section, the different types of cooperation are compared in terms of R&D, welfare, and profits. Before proceeding with the analysis, it will be helpful to restate that the advertising spillovers may be positive or negative. The negative advertising spillovers have a negative impact on the rival's demand. Consequently, a cooperation setting which internalizes this negative effect yields lower R&D. However, internalizing positive advertising spillovers may increase (decrease) innovation depending on spillovers and product substitutability. Moreover, when R&D and advertising spillovers are low, the private gain from R&D and advertising is substantial and boosts firms' incentive to increase them. This may induce firms to overinvest in R&D when they choose R&D and advertising non-cooperatively due to intense competition. Additionally, under no cooperation, the importance of outgoing spillovers may induce firms to underinvest in R&D when spillovers are sufficiently high.

The following Proposition summarizes the ranking of cooperation settings in terms of R&D, profits and welfare.

Proposition 4.

(i) *If* $\theta < 0, \beta = 0$ *and* $b = 0$, *then* $X_{RN} = X_{NN} > X_{NA} = X_{RA}$ *and* $W_{RN} = W_{NN} > W_{NA} = W_{RA}$; *however,* $\pi_{RA} = \pi_{NA} > \pi_{RN} = \pi_{NN}$

(ii) *If* $\theta < 0, 0 < \beta < 0.1$ *and* $b = 0$, *then* $X_{RN} > X_{NN} > X_{RA} > X_{NA}$ *and* $W_{RN} > W_{NN} > W_{RA} > W_{NA}$; *however,* $\pi_{RA} > \pi_{NA} > \pi_{RN} > \pi_{NN}$

(iii) *If* $\theta < 0, \beta > 0.1$ *and* $b = 0$, *then* $X_{RN} > X_{RA} > X_{NN} > X_{NA}$ *and* $W_{RN} > W_{RA} > W_{NN} > W_{NA}$; *however,* $\pi_{RA} > \pi_{NA} > \pi_{RN} > \pi_{NN}$

(iv) *If* $\theta < -b/2, \beta < b/2$ *and* $b > 0$, *then* $X_{NN} > X_{RN} > X_{NA} > X_{RA}$ *and* $W_{NN} > W_{RN} > W_{NA} > W_{RA}$; *however,* $\pi_{RA} > \pi_{NA} > \pi_{RN} > \pi_{NN}$

(v) *If* $\theta < -b/2$, $\beta > b/2$ *and* $b > 0$, *then* $X_{RN} > X_{RA} > X_{NN} > X_{NA}$ *and* $W_{RN} > W_{RA} > W_{NN} > W_{NA}$; *however,* $\pi_{RA} > \pi_{NA} > \pi_{RN} > \pi_{NN}$

(vi) *If* $\theta < b/2$ *and* $\beta < b/2$ *where* $\theta < \beta$, *then* $X_{NN} > X_{RN} > X_{NA} > X_{RA}$ *and* $W_{NN} > W_{RN} > W_{NA} > W_{RA}$; *however,* $\pi_{RA} > \pi_{NA} > \pi_{RN} > \pi_{NN}$

(vii) *If* $\theta < b/2$ *and* $\beta < b/2$ *where* $\theta > \beta$, *then* $X_{NN} > X_{NA} > X_{RN} > X_{RA}$ *and* $W_{NN} > W_{NA} > W_{RN} > W_{RA}$; *however,* $\pi_{RA} > \pi_{NA} > \pi_{RN} > \pi_{NN}$

(viii) *If* $\theta > b/2$ *and* $\beta > b/2$ *where* $\theta < \beta$*, then* $X_{RA} > X_{RN} > X_{NA} > X_{NN}$ *and* $W_{RA} > W_{RN} > W_{NA} > W_{NN}$*; however,* $\pi_{RA} > \pi_{NA} > \pi_{RN} > \pi_{NN}$

(ix) *If* $\theta > b/2$ *and* $\beta > b/2$ *where* $\theta > \beta$*, then* $X_{RA} > X_{NA} > X_{RN} > X_{NN}$ *and* $W_{RA} > W_{NA} > W_{RN} > W_{NN}$*; however,* $\pi_{RA} > \pi_{NA} > \pi_{RN} > \pi_{NN}$

To better understand the ranking of cooperation settings, it is helpful to review the competitive externalities and the effects of internalizing them. Cooperative R&D induces firms to internalize the effect of R&D on the competitor's profit. In other words, the competitive externality associated with R&D reflects the marginal impact of the R&D of a firm on the profits of other firms. In this model there are two types of competitive externalities: the advertising externality (*AE*)[6] and the R&D externality (*RE*).[7] *AE* represents the marginal effect of a firm's advertising on the competitor's profit. This externality is internalized under *NA* and *RA*. *AE* can be positive or negative, depending on whether an increase in the advertising of a firm increases or decreases the rival's profit. *AE* increases with spillovers, advertising and R&D of the competitor and decreases with product substitutability. *RE* represents the marginal effect of a firm's R&D on the rival's profit. This externality is internalized under *RN* and *RA*.[8] *RE* can be positive or negative, depending on whether an increase in R&D by a firm increases or decreases the rival's profit. *RE* increases with spillovers, advertising and R&D of the competitor and decreases with product substitutability. No competitive externalities are internalized under *NN*.

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[6] $AE = \frac{\partial \pi_1}{\partial a_2} = \frac{2(2\theta - b)[(k-\alpha)+(1+\theta)a_2+(1+\beta)x_2]}{(2-b)(2+b)^2}$.

[7] $RE = \frac{\partial \pi_1}{\partial x_2} = \frac{2(2\beta - b)[(k-\alpha)+(1+\theta)a_2+(1+\beta)x_2]}{(2-b)(2+b)^2}$.

[8] *RA* internalizes the sum of *AE* and *RE*, such that $\frac{\partial \pi_1}{\partial x_2} + \frac{\partial \pi_1}{\partial a_2} = \frac{4(\theta+\beta-b)[(k-\alpha)+(1+\theta)a_2+(1+\beta)x_2]}{(2-b)(2+b)^2}$.

First we compare R&D across the different settings, then we discuss the relationship with profits and welfare.

Part *i* of Proposition 4 states that $X_{NN} = X_{RN} > X_{NA} = X_{RA}$ if θ is negative, β is nil and goods are independent. Competition between firms is affected by advertising and R&D. When θ is negative, *AE* is negative. Internalizing the negative advertising externality reduces the net benefit of advertising, reducing firms' incentive to increase output and R&D. *NA* internalizes the negative *AE*. Additionally, when $\beta = 0$, internalizing *RE* is not effective since *RE* is nil. *RN* internalizes *RE* which is not effective (since $\beta = 0$ and $b = 0$), while *RA* internalizes the sum of *AE* and *RE* which is negative. Under *NN*, no competitive externality is internalized. Thus, *NN* and *RN* yield the highest level of R&D. *NA* and *RA* yield the same level of R&D which is lower than under *RN* and *NN*.

Part *ii* of Proposition 4 states that $X_{RN} > X_{NN} > X_{RA} > X_{NA}$ when θ is negative, β is sufficiently low, and goods are independent. The negative θ is associated with negative externalities. When β is sufficiently low, its externality is low yet positive since $\beta > 0$. *RN* internalizes *RE*, whereas *RA* internalizes the sum of *RE* and *AE*. Although firms may weakly benefit from internalizing *RE*, internalizing the negative advertising externalities (*RE* + *AE* < 0) more than offsets the benefits. Thus, the highest innovation is achieved under *RN*. *NN* internalizes no externality and ranks second. *NA* just internalizes *AE* which is negative and yields lower R&D than *RA*.

Part *iii* of Proposition 4 states that $X_{RN} > X_{RA} > X_{NN} > X_{NA}$ when θ is negative, β is intermediate, and goods are independent. The negative θ is associated with negative *AE*, whereas β is associated with positive *RE*. *RN* internalizes the positive *RE*. Thus, the highest R&D is achieved under *RN*. *RA* internalizes the sum of *RE* and *AE* which is positive (*RE* + *AE* > 0), yet it is less than *RE*. In fact, the effects of negative *AE* mitigate the effects of the positive *RE* and thereby, *RA* ranks second. Comparing *NN* and *NA*, *NA* internalizes the negative *AE* which reduces firms' incentive to innovate, while *NN* internalizes no externality. Conclusively, *NN* yields higher R&D than *NA*.

Part *iv* of Proposition 4 states that $X_{NN} > X_{RN} > X_{NA} > X_{RA}$ when θ is negative, β is sufficiently low, and goods are substitutes. When goods are substitutes, competition is intense. Thus, firms are worried about the effects of their R&D and advertising on the rival through spillovers. The negative θ and the sufficiently low β are both associated with negative competitive externalities when $b > 0$. However, the negative effects of *AE* are stronger than the effects of *RE*. *NN* internalizes no externality, yields the highest innovation. *RN* ranks second by internalizing the weakly negative *RE*. *NA* internalizes the negative externality of advertising (*AE* > *RE*) and ranks third. *RA* internalizes the sum of the negative competitive externalities and yields the least R&D.

Part *v* of Proposition 4 states that $X_{RN} > X_{RA} > X_{NN} > X_{NA}$ when θ is negative, β is high and goods are substitutes. Although the negative θ is associated with negative competitive externalities, β is associated with positive externality if it is high enough. Thus, internalizing *RE* benefits firms and induces them to increase R&D. *RN* internalizes the positive competitive externality of R&D and increases the net benefit of R&D. *RA* internalizes the sum of *AE* and *RE* where the negative effects of *AE* partially offset the positive effect of *RE*. Thus, the highest innovation is achieved under *RN*, whereas the second rank is achieved under *RA*. Comparing *NN* and *NA* shows that *NN*, which internalizes no externality, yields higher R&D than *NA* which internalizes the negative advertising spillovers.

Part *vi* of Proposition 4 states that $X_{NN} > X_{RN} > X_{NA} > X_{RA}$ when θ and β are positive and sufficiently low, yet $\beta > \theta$. When θ and β are sufficiently low, they are associated with negative competitive externalities. Since θ is less than β, the negative effects of internalizing θ dominate the effects of internalizing β. Thus, *RA* yields the least innovation by internalizing the sum of these negative externalities. However, the highest innovation is achievable under *NN* since *NN* internalizes no externality. Comparing *RN* and *NA* where $\beta > \theta$ shows that *RN* yields higher innovation than *NA*.

Part *vii* of Proposition 4 states that $X_{NN} > X_{NA} > X_{RN} > X_{RA}$ when θ and β are positive and sufficiently low, yet $\theta > \beta$. Note that θ and β are associated with negative competitive externalities when they are sufficiently low. Internalizing any negative externality reduces the net benefit of R&D and induces firms to reduce R&D. The negative competitive externality of advertising is less than the negative competitive externality of R&D when $\theta > \beta$. Therefore, *NA* yields higher innovation by internalizing θ than *RN* internalizing β. Indeed, *RA* yields the least innovation by internalizing the sum of these negative externalities. However, the highest innovation is achieved by *NN*, which does not internalize any externality.

Part *viii* of Proposition 4 states that $X_{RA} > X_{RN} > X_{NA} > X_{NN}$ when θ and β are positive and high yet $\beta > \theta$. When θ and β are high enough, they are associated with positive competitive externalities. Internalizing the positive competitive externalities benefits firms, increases the net benefit of R&D, and induces firms to increase output and R&D. *RA* internalizes the sum of positive competitive externalities of advertising and R&D. Thus, *RA* yields the highest innovation. Comparing *RN* and *NA* shows that although both internalize a positive competitive externality, *RN* yields higher innovation than *NA* since *RE* > *AE*. Moreover, *NN* yields the least innovation by internalizing no externality.

Part *ix* of Proposition 4 states that $X_{RA} > X_{NA} > X_{RN} > X_{NN}$ when θ and β are positive and high, yet $\theta > \beta$. Internalizing positive externalities increases the net benefit of R&D and induces firms to increase innovation. Thus, *RA* yields the highest innovation by internalizing the sum of positive competitive externalities. Indeed, *NA* internalizes a larger positive competitive externality than *RN*, since *AE* > *RE*. Thus, *NA* ranks second and *RN*, third. Finally, *NN* internalizes no externality and yields the least innovation.

Higher R&D reduces production cost and yields higher output, which yields higher consumer surplus and welfare. Thus, the ranking for cooperation settings in terms of welfare is the same as the ranking in terms of R&D. However, the results show that firms always gain the highest profit under *RA*. *NA* yields higher profit than *RN* regardless of spillovers and substitutability. This is due to the impact of advertising cooperation on market size. Cooperative advertising mitigates the negative effects of the negative advertising spillovers when θ is negative, while it reinforces the positive effects when θ is positive. The lowest profit is achieved under *NN*. The results show that when goods are substitutes and spillovers are sufficiently large, $\theta > \beta$, the ranking of cooperation settings in terms of welfare and profit are in line (cases *viii* and *ix* of Proposition 4). Otherwise, there is a market failure, in that what firms will choose yields suboptimal innovation and welfare (compared to at least one of the other three settings).

This analysis suggests that full R&D and advertising cooperation is always preferred by firms to partial cooperation or no cooperation. When R&D and advertising spillovers are high enough, this leads to maximum innovation and welfare (in a second best setting).[9] In this situation, the private incentives of firms coincide with consumer and social welfare. When, however, these conditions are not met, so that either advertising spillovers are low (or negative),

[9] This situation corresponds to the last two cases of proposition 4 above, as already mentioned.

and/or R&D spillovers are sufficiently low, then this full cooperation tends to reduce innovation and welfare, and also harms consumers.[10] Actually, in these circumstances full R&D and advertising cooperation yields a dismal performance on innovation and welfare, ranking third or fourth among the four possible cooperative settings. In this case, there is a clear and severe market failure. Thus, governments should encourage R&D and advertising cooperation when R&D spillovers and advertising spillovers are sufficiently high. Otherwise, these types of cooperation should be met with antitrust scrutiny.

7. Conclusion

This paper aims to study the impact of cooperative R&D and advertising on innovation and welfare by duopolists producing differentiated products in the presence of R&D and advertising spillovers. Firms choose R&D and advertising in the first stage and compete in output in the second stage. Four types of cooperation are considered: no cooperation, R&D cooperation, advertising cooperation, and finally joint R&D and advertising cooperation. The effects of cooperation strategies, product differentiation, and spillovers on innovation, advertising, output, welfare, and profit are considered. Additionally, strategic interaction between R&D and advertising is studied. Finally, cooperation settings are compared in terms of R&D, welfare, and profits.

The comparison in terms of innovation and welfare shows that no type of cooperation uniformly dominates the others. The type of cooperation yielding more innovation depends on R&D spillovers, advertising spillovers, and product differentiation. The ranking of cooperative structures rests on R&D and advertising externalities which capture the effect of the R&D and advertising of a firm on the rival’s profit. The cooperative structure inducing firms to internalize

[10] This corresponds to the first seven cases of proposition 4.

higher positive competitive externalities yields higher innovation. The no cooperation setting dominates the other settings when spillovers are sufficiently low. Cooperative R&D and advertising yield the highest innovation when spillovers are sufficiently high. However, firms always prefer cooperating in advertising and R&D simultaneously.

Strategic interaction among firms shows that innovation and advertising are strategic complements when spillovers are high enough and/or product substitutability is low enough. Innovation and advertising are strategic substitutes when spillovers are low and/or product substitutability is sufficiently high.

The impact of R&D spillovers on innovation, output, profit, and welfare are positive when firms cooperate in R&D. However, the impact on innovation, output, profit, and welfare depends on spillovers and product substitutability when firms do not cooperate in R&D. Additionally, the effects of advertising spillovers on innovation, output and welfare are always positive when firms cooperate in advertising. However, the effect on innovation, output, profit, and welfare depends on spillovers and product substitutability when firms do not cooperate in advertising.

One of the basic results of the strategic cooperative R&D and advertising literatures is the effect of cost sharing between competitors which enables firms to invest in more advertising and R&D, and positively affects innovation through internalizing spillovers when they are high enough. However, the model shows that these results do not necessarily hold when advertising and/or R&D spillovers are sufficiently low. In particular, cooperative advertising between competitors decreases R&D when advertising spillovers are negative. Moreover, when R&D spillovers are high enough, yet advertising spillovers are negative (sufficiently low), R&D

cooperation still yields higher innovation. Indeed, firms attain the highest profits by cooperating in R&D and advertising regardless of spillovers and substitutability.

The results of this paper can be tested empirically. Effects of spillovers and product substitutability on innovation, output, and welfare, the effect of collusion on innovation and advertising, and how firms respond to different R&D and advertising strategies of their competitors can be addressed by empirical studies.

The study of cooperative R&D and of cooperative advertising raises important science, technology, and marketing policy issues. Since this model abstracts from many real world issues, especially asymmetric information between the regulators and firms, drawing policy recommendations from the model requires carefulness. The model provides some considerations of R&D policy and advertising regulation from the point of view of the incentives of cooperation and mergers.

The model points to an interaction between R&D cooperation policy, patent policy, and advertising cooperation procedures. To take advantage of internalizing the beneficial advertising and technological spillovers where firms are allowed to cooperate, a slack patent and advertising policy is recommended. However, innovation should be protected where cooperation is prohibited.

The model shows that the optimal R&D and advertising policy varies according to R&D spillovers, advertising spillovers, product substitutability and the prevailing cooperative structure. The model suggests that R&D cooperation is more often socially beneficial than advertising cooperation when advertising spillovers are negative. If, however, advertising spillovers are positive the cooperative structure which internalizes the most positive competitive externalities is more socially beneficial.

The results have implications for merger analysis. To maximize joint profits, mergers usually require the use of R&D and marketing. Economists usually focus on the output and efficiency effects of mergers, nevertheless, the innovation effects of mergers require more attention. The results show that the innovation effects of mergers in duopolistic industries depend on R&D and advertising spillovers. The appropriability conditions, absorptive capacities, the prevailing cooperative structure, spillovers, product substitutability, and market structure before the merger, are the key elements that lead the merger to reinforce or mitigate the negative effect of output reduction by increasing or decreasing R&D and/or advertising. In other words, this analysis points toward strict merger policy where output decisions are joint, yet R&D decisions and/or advertising decisions remain separate.

In terms of equilibrium analysis, firms always prefer full R&D and advertising cooperation. This combination yields maximum innovation and welfare when both advertising spillovers and R&D spillovers are sufficiently high. Otherwise, when either of these spillovers are low enough, this full cooperation yields quite inferior outcomes in terms of innovation and welfare, and should be analyzed thoroughly by antitrust authorities.

The model has many possible extensions. A critical determinant of cooperative R&D and advertising is the level of spillovers, which may be affected by firm size, which has not been addressed in this paper. It was assumed that the companies are symmetric and active in the same industry. In the real world, firm size, absorptive capacity, the pace and scope of technological change, and communication channels are different among firms (industries) which leads to different levels of technological knowledge and affects firm marketing strategies. This, in turn, may affect the symmetry of firm behaviour.

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APPENDIX

A.1 Proof of Proposition 1

$$\pi_i = \frac{[(2-b)(k-\alpha)-b\theta a_i-b\beta x_i+2(a_i+x_i)+a_j(2\theta-b)+x_j(2\beta-b)]^2}{(4-b^2)^2} - \frac{1}{2}\lambda a_i^2 - \frac{1}{2}\gamma x_i^2 \qquad i,j=1,2 \qquad i\neq j \qquad \text{(A.1)}$$

Taking the derivative of Eq. (A.1) with respect to own R&D and advertising and the competitor's to evaluate the strategic interaction shows that:

(1) $\frac{\partial^2 \pi_i}{\partial x_i \partial x_j} > 0$ if $(2\beta - b) > 0$; otherwise, it is negative

(2) $\frac{\partial^2 \pi_i}{\partial x_i \partial a_j} > 0$ if $(2\theta - b) > 0$; otherwise, it is negative.

(3) $\frac{\partial^2 \pi_i}{\partial a_i \partial x_j} > 0$ if $(2\beta - b) > 0$; otherwise, it is negative.

(4) $\frac{\partial^2 \pi_i}{\partial a_i \partial a_j} > 0$ if $(2\theta - b) > 0$; otherwise, it is negative.

A.2 Proof of Proposition 2

$$\frac{\partial X^{NN}}{\partial \beta} = \frac{4\lambda(k-\alpha)((2(2-b\beta)^2-2b\gamma(1+\theta)(2-b\theta)+b\gamma\lambda(2-b)(2+b)^2))}{(2\gamma(1+\theta)(2-b\theta)+2\lambda(1+\beta)(2-b\beta)-\gamma\lambda(2-b)(2+b)^2)^2} \qquad \text{(A.2)}$$

$$\frac{\partial a^{NN}}{\partial \beta} = \frac{8\gamma(k-\alpha)(2-b-2b\beta)(2-b\theta)}{(\gamma(2-b)(2+b)^2-2(1+\beta)(2-b\beta)-2\gamma(1+\theta)(2-b\theta))^2} \qquad \text{(A.3)}$$

$$\frac{\partial \pi^{NN}}{\partial \beta} = \frac{(4\gamma\lambda^2(k-\alpha)^2[2\gamma(2-b\theta)\big(4+b\big(b\theta(1+\beta)-\beta(4+b)\big)\big)+2\lambda(2-b\beta)^3+\gamma\lambda(2-b)(2+b)^2\big(4+b\big(2-b+\beta(4-b)\big)\big)]}{(2\gamma(1+\theta)(2-b\theta)+2\lambda(1+\beta)(2-b\beta)-\gamma\lambda(2-b)(2+b)^2)^3}$$

(A.4)

$$\frac{\partial Y^{NN}}{\partial \beta} = \frac{4\gamma(4-b^2)(k-\alpha)(2-b-2b\beta)}{(2(1+\beta)(2-b\beta)-\gamma(2-b)(2+b)^2+2\gamma(1+\theta)(2-b\theta))^2} \qquad \text{(A.5)}$$

$$\frac{\partial W^{NN}}{\partial \beta}=\frac{4\gamma\lambda^2(k-\alpha)^2[4\gamma(2-b\theta)\Big(4+b\big(b\theta(1+\beta)-\beta(4+b)\big)\Big)+4\lambda(2-b\beta)^3-\gamma\lambda(2-b)(2+b)^2(12-b\Big(4+b-b^2+2\beta\big(6-b^2\big)\Big)]}{2\gamma(1+\theta)(2-b\theta)+2\lambda(1+\beta)(2-b\beta)-\gamma\lambda(2-b)(2+b)^2)^3} \tag{A.6}$$

$$p^{NN}=\frac{2\alpha\gamma(1+\theta)(2-b\theta)+\lambda(2k(1+\beta)(2-b\beta)-\gamma(4-b^2)(k+\alpha+b\alpha))}{2\gamma(1+\theta)(2-b\theta)+2\lambda(1+\beta)(2-b\beta)-\gamma\lambda(2-b)(2+b)^2} \tag{A.7}$$

$$\frac{\partial X^{RN}}{\partial \beta}=\frac{4\lambda(2-b)(k-\alpha)[\lambda(2-b)\big(2(1+\beta)^2+\gamma(2+b)^2\big)-2\gamma(1+\theta)(2-b\theta)]}{(2\gamma(1+\theta)(2-b\theta)+\lambda(2-b)(2(1+\beta)^2-\gamma(2+b)^2)^2} \tag{A.8}$$

$$\frac{\partial a^{RN}}{\partial \beta}=\frac{16\gamma(2-b)(k-\alpha)(1+\beta)(2-b\theta)}{((2-b)[2(1+\beta)^2-\gamma(2+b)^2]-2\gamma(1+\theta)(2-b\theta))^2} \tag{A.9}$$

$$\frac{\partial \pi^{RN}}{\partial \beta}=\frac{(4\gamma\lambda^2(2-b)(k-\alpha)^2(1+\beta)[2\gamma(2+b)(1-\theta)(2-b\theta)+\lambda(2-b)^2\big(2(1+\beta)^2-\gamma(2+b)^2\big)]}{(2\gamma(1+\theta)(2-b\theta)+\lambda(2-b)[2(1+\beta)^2-\gamma(2+b)^2])^3} \tag{A.10}$$

$$\frac{\partial Y^{RN}}{\partial \beta}=\frac{8\gamma(2-b)^2(2+b)(k-\alpha)(1+\beta)}{((2-b)(2(1+\beta)^2-\gamma(2+b)^2)+2\gamma(1+\theta)(2-b\theta))^2} \tag{A.11}$$

$$\frac{\partial W^{RN}}{\partial \beta}=\frac{8\gamma\lambda^2(2-b)(k-\alpha)^2(1+\beta)[2\lambda(2-b)^2(1+\beta)^2+\gamma(2+b)(2(1-\theta)(2-b\theta)-\lambda\big(4-b^2\big)^2]}{(2\gamma(1+\theta)(2-b\theta)+\lambda(2-b)(2(1+\beta)^2-\gamma(2+b)^2))^3} \tag{A.12}$$

$$p^{RN}=\frac{(k-\alpha)^2\gamma\lambda(\lambda(2-b)^2(\gamma(2+b)^2-2(1+\beta)^2)-2\gamma(2-b\theta)^2)}{(2\gamma(1+\theta)(2-b\theta)+\lambda(2-b)(2(1+\beta)^2-\gamma(2+b)^2))^2} \tag{A.13}$$

$$\frac{\partial X^{NA}}{\partial \beta}=\frac{4\lambda(k-\alpha)(2b\gamma(2-b)(1+\theta)^2+\lambda[2(2-b\beta)^2-b\gamma(2-b)(2+b)^2])}{(2\lambda(1+\beta)(2-b\beta)+\gamma(2-b)[2(1+\theta)^2-\lambda(2+b)^2])^2} \tag{A.14}$$

$$\frac{\partial a^{NA}}{\partial \beta}=\frac{8\gamma(2-b)(k-\alpha)(2-b-2b\beta)(1+\theta)}{(2(1+\beta)(2-b\beta)+\gamma(2-b)(2(1+\theta)^2-(2+b)^2))^2} \tag{A.15}$$

$$\frac{\partial \pi^{NA}}{\partial \beta}=\frac{4\gamma\lambda^2(k-\alpha)^2(2\lambda(2-b\beta)^3+\gamma(2-b)(4-b(2-b+\beta(4-b)))(2(1+\theta)^2-\lambda(2+b)^2))}{(2\lambda(1+\beta)(2-b\beta)+\gamma(2-b)(2(1+\theta)^2-\lambda(2+b)^2))^3} \tag{A.16}$$

$$\frac{\partial Y^{NA}}{\partial \beta}=\frac{4\gamma(4-b^2)(k-\alpha)(2-b-2b\beta)}{(2(1+\beta)(2-b\beta)+\gamma(2-b)(2(1+\theta)^2+(2+b)^2))^2} \tag{A.17}$$

$$\frac{\partial W^{NA}}{\partial \beta}=\frac{4\gamma\lambda^2(k-\alpha)^2(4\lambda(2+b\beta)^3+\gamma(2-b)[4\Big(4-b\big(2-b-\beta(4-b)\big)\Big)(1+\theta)^2-\lambda(2+b)^2(12-b\Big(4+b-b^2+2\beta\big(6-b^2\big)\Big)])}{(2\lambda(1+\beta)(2-b\beta)+\gamma(2-b)(2(1+\theta)^2-\lambda(2+b)^2))^3} \tag{A.18}$$

$$p^{NA}=\frac{(2\alpha\gamma(2-b)(1+\theta)^2-(2k(1+\beta)(2-b\beta)+(4-b^2)(k+\alpha+b\alpha)\gamma)\lambda)}{(2\lambda(1+\beta)(2-b\beta)+\gamma(2-b)(2(1+\theta)^2-\lambda(2+b)^2))} \tag{A.19}$$

$$\frac{\partial X^{RA}}{\partial \beta} = \frac{4\lambda(k-\alpha)(2\lambda(1+\beta)^2-2\gamma(1+\theta)^2+\gamma\lambda(2+b)^2)}{(2\gamma(1+\theta)^2+2\lambda(1+\beta)^2-\gamma\lambda(2+b)^2)^2} \tag{A.20}$$

$$\frac{\partial a^{RA}}{\partial \beta} = \frac{16\gamma(k-\alpha)(1+\beta)(1+\theta)}{(2\gamma(1+\theta)^2+2(1+\beta)^2-(2+b)^2\gamma)^2} \tag{A.21}$$

$$\frac{\partial \pi^{RA}}{\partial \beta} = \frac{4\gamma(k-\alpha)^2(1+\beta)}{(2\gamma(1+\theta)^2+2(1+\beta)^2-\gamma(2+b)^2)^2} \tag{A.22}$$

$$\frac{\partial Y^{RA}}{\partial \beta} = \frac{8\gamma(k-\alpha)(2+b)(1+\beta)}{(2\gamma(1+\theta)^2+2(1+\beta)^2-(2+b)^2\gamma)^2} \tag{A.23}$$

$$\frac{\partial W^{RA}}{\partial \beta} = \frac{8\gamma\lambda^2(k-\alpha)^2(1+\beta)(2\gamma(1+\theta)^2+2\lambda(1+\beta)^2-\gamma\lambda(2+b)^3)}{(2\gamma(1+\theta)^2+2\lambda(1+\beta)^2-\gamma\lambda(2+b)^2)^3} \tag{A.24}$$

$$p^{RA} = \frac{2\alpha\gamma(1+\theta)^2+2k(1+\beta)^2-\gamma(2+b)(k+\alpha+b\alpha)}{2\gamma(1+\theta)^2+2(1+\beta)^2-\gamma(2+b)^2} \tag{A.25}$$

Note that to simplify the analysis, λ is fixed at $\lambda = 1$ by assumption. Output is positive if $k > \alpha$. Considering $k = \alpha$ & $\lambda = 1$ and evaluating p at different parameter values indicates the validity of γ and shows that it meets the required thresholds where required.

Evaluating the above equations for any value of parameters shows that R&D spillovers have a positive impact on innovation, output, profit, and welfare when firms cooperate in R&D. Otherwise, the impact is positive if R&D spillovers are low enough.

A.3 Proof of Proposition 3

$$\frac{\partial X^{NN}}{\partial \theta} = \frac{8\gamma(k-\alpha)(2-b\beta)(2-b-2b\theta)}{(2\gamma(1+\theta)(2-b\theta)+2(1+\beta)(2-b\beta)-(2-b)(2+b)^2\gamma)^2} \tag{A.26}$$

$$\frac{\partial a^{NN}}{\partial \theta} = \frac{4\gamma(k-\alpha)(2\gamma(2-b\theta)^2+2\lambda b(1+\beta)(2-b\beta)-b\gamma\lambda(2-b)(2+b)^2)}{(2\gamma(1+\theta)(2-b\theta)+2\lambda(1+\beta)(2-b\beta)+\gamma\lambda(2-b)(2+b)^2)^2} \tag{A.27}$$

$$\frac{\partial \pi^{NN}}{\partial \theta} = \frac{4\gamma^2\lambda(k-\alpha)^2[2\gamma(2-b\theta)^3-\lambda\gamma(2-b)(2+b)^2\big(4-b\big(2-b+\theta(4-b)\big)\big)+2\lambda(2-b\beta)(4-4b\theta+b^2\big(\beta-\theta(1-\beta)\big))]}{(2\gamma(1+\theta)(2-b\theta)+2\lambda(1+\beta)(2-b\beta)-\gamma\lambda(2-b)(2+b)^2)^3}$$

(A.28)

$$\frac{\partial Y^{NN}}{\partial \theta} = \frac{4\gamma^2(4-b^2)(k-\alpha)(2-b-2b\theta)}{(2\gamma(1+\theta)(2-b\theta)+2(1+\beta)(2-b\beta)-\gamma(2-b)(2+b)^2)^2} \tag{A.29}$$

$$\frac{\partial W^{NN}}{\partial \theta} = \frac{4\gamma^2\lambda(k-\alpha)^2[4\gamma(2-b\theta)^3-\gamma\lambda(2-b)(2+b)^2\left(12-b\left(4+b-b^2-2\theta(6-b^2)\right)\right)+4\lambda(2-b\beta)(4-4b\theta+b^2(\beta-\theta(1-\beta)))]}{(2\gamma(1+\theta)(2-b\theta)+2\lambda(1+\beta)(2-b\beta)-\gamma\lambda(2-b)(2+b)^2)^3} \tag{A.30}$$

$$\frac{\partial X^{RN}}{\partial \theta} = \frac{8\gamma(2-b)(k-\alpha)(1+\beta)(2-b-2b\theta)}{\left(2\gamma(1+\theta)(2-b\theta)+(2-b)(2(1+\beta)^2+\gamma(2+b)^2)\right)^2} \tag{A.31}$$

$$\frac{\partial a^{RN}}{\partial \theta} = \frac{4\gamma(k-\alpha)(2\gamma(2-b\theta)^2+\lambda b(2-b)(2(1+\beta)^2-\gamma(2+b)^2))}{(2\gamma(1+\theta)(2-b\theta)+\lambda(2-b)(2(1+\beta)^2-\gamma(2+b)^2))^2} \tag{A.32}$$

$$\frac{\partial \pi^{RN}}{\partial \theta} = \frac{4\gamma^2\lambda(k-\alpha)^2[2\gamma(2-b\theta)^3+\lambda(2-b)\left(2(1+\beta)^2-\gamma(2+b)^2\right)\left(4-b\left(2-b-\theta(4-b)\right)\right)]}{(2\gamma(1+\theta)(2-b\theta)+\lambda(2-b)(2(1+\beta)^2-\gamma(2+b)^2))^3} \tag{A.33}$$

$$\frac{\partial Y^{RN}}{\partial \theta} = \frac{4\gamma^2(4-b^2)(k-\alpha)(2-b-2b\theta)}{(2\gamma(1+\theta)(2-b\theta)+(2-b)(2(1+\beta)^2-\gamma(2+b)^2))^2} \tag{A.34}$$

$$\frac{\partial W^{RN}}{\partial \theta} = \frac{4\gamma^2\lambda(k-\alpha)^2[4\gamma(2-b\theta)^3+4\lambda(2-b)(1+\beta)^2\left(4-b\left(2-b+\theta(4-b)\right)\right)-\lambda\gamma(2-b)(2+b)^2\left(12-b\left(4+b-b^2-2\theta(6-b^2)\right)\right)]}{(2\gamma(1+\theta)(2-b\theta)+\lambda(2-b)(2(1+\beta)^2-(2+b)^2\gamma))^3} \tag{A.35}$$

$$\frac{\partial X^{NA}}{\partial \theta} = \frac{16\gamma(2-b)(k-\alpha)(2-b\beta)(1+\theta)}{(2(1+\beta)(2-b\beta)+\gamma(2-b)(2(1+\theta)^2-(2+b)^2))^2} \tag{A.36}$$

$$\frac{\partial a^{NA}}{\partial \theta} = \frac{4\gamma(2-b)(k-\alpha)(\gamma(2-b)(2(1+\theta)^2+\lambda(2+b)^2)-2\lambda(1+\beta)(2-b\beta))}{(2\lambda(1+\beta)(2-b\beta)+\gamma(2-b)(2(1+\theta)^2-\lambda(2+b)^2))^2} \tag{A.37}$$

$$\frac{\partial \pi^{NA}}{\partial \theta} = \frac{4\gamma^2\lambda(2-b)(k-\alpha)^2[(1+\theta)(2\lambda(2+b)(1-\beta)(2-b\beta)+\gamma(2-b)^2\left(\lambda(2+b)^2-2(1+\theta)^2\right)]}{(2\lambda(1+\beta)(2-b\beta)+\gamma(2-b)(2(1+\theta)^2-\lambda(2+b)^2))^3} \tag{A.38}$$

$$\frac{\partial Y^{NA}}{\partial \theta} = \frac{8\gamma^2(k-\alpha)(1+\theta)(2-b)^2(2+b)}{(2(1+\beta)(2-b\beta)+\gamma(2-b)(2(1+\theta)^2-(2+b)^2))^2} \tag{A.39}$$

$$\frac{\partial W^{NA}}{\partial \theta} = \frac{8\gamma^2\lambda(2-b)(k-\alpha)^2(1+\theta)[2\lambda(2+b)(1-\beta)(2-b\beta)-\gamma(2-b)^2\left(2(1+\theta)^2-\lambda(2+b)^3\right)]}{(2\lambda(1+\beta)(2-b\beta)+\gamma(2-b)(2(1+\theta)^2-\lambda(2+b)^2))^3} \tag{A.40}$$

$$\frac{\partial X^{RA}}{\partial \theta} = \frac{16(k-\alpha)(1+\beta)\gamma(1+\theta)}{(2\gamma(1+\theta)^2+2(1+\beta)^2-\gamma(2+b)^2)^2} \tag{A.41}$$

$$\frac{\partial a^{RA}}{\partial \theta} = \frac{4\gamma(k-\alpha)(2\gamma(1+\theta)^2-2(1+\beta)^2\lambda+(2+b)^2\gamma\lambda)}{(2\gamma(1+\theta)^2+2(1+\beta)^2\lambda-(2+b)^2\gamma\lambda)^2} \quad \text{(A.42)}$$

$$\frac{\partial \pi^{RA}}{\partial \theta} = \frac{4\gamma^2(k-\alpha)^2(1+\theta)}{(2\gamma(1+\theta)^2+2(1+\beta)^2-\gamma(2+b)^2)^2} \quad \text{(A.43)}$$

$$\frac{\partial Y^{NA}}{\partial \theta} = \frac{8\gamma^2(2+b)(k-\alpha)(1+\theta)}{(2\gamma(1+\theta)^2+2(1+\beta)^2-\gamma(2+b)^2)^2} \quad \text{(A.44)}$$

$$\frac{\partial W^{RA}}{\partial \theta} = \frac{8\gamma^2\lambda(k-\alpha)^2(1+\theta)[2\gamma(1+\theta)^2+2\lambda(1+\beta)^2-\gamma\lambda(2+b)^3]}{(2\gamma(1+\theta)^2+2\lambda(1+\beta)^2-\gamma\lambda(2+b)^2)^3} \quad \text{(A.45)}$$

Evaluating the above equations for any value of parameters shows that advertising spillovers have a positive impact on innovation, output, profit, and welfare when firms cooperate in advertising. Otherwise, the impact is positive if advertising spillovers are low enough.

A.4 Proof of Proposition 4

Section 3 provides the formula for x_i, π_i, and W as a function of other parameters. Taking the deduction of each pair of equations shows that for any parameter set that yields a valid price value ($p > 0$) the following holds.

(1) If $b = \beta = 0$ and $\theta < 0$, then $X_{NN} = X_{RN} > X_{NA} = X_{RA}$, $\pi_{NN} = \pi_{RN} > \pi_{NA} = \pi_{RA}$, and $W_{NN} = W_{RN} > W_{NA} = W_{RA}$

(2) If $b = 0$, $0 < \beta < 0.1$, and $\theta < 0$, then $X_{RN} > X_{NN} > X_{RA} > X_{NA}$ and $W_{RN} > W_{NN} > W_{RA} > W_{NA}$.

(3) If $b = 0$ and $\beta > 0.1$, and $\theta < 0$, then $X_{RN} > X_{RA} > X_{NN} > X_{NA}$ and $W_{RN} > W_{RA} > W_{NN} > W_{NA}$.

(4) $X_{NA} > X_{RA}$ if $\theta < -b/2$ and $\beta < b/2$

(5) $X_{RN} > X_{NA}$ if $\theta < -b/2$ and $\beta < b/2$

(6) $X_{RA} > X_{RN}$ if $\theta > b/2$

(7) $X_{NA} > X_{NN}$ if $\theta > b/2$

(8) $X_{RN} > X_{NN}$ if $\beta > b/2$

(9) $X_{RN} > X_{NA}$ if $\theta < \beta$

(10) $X_{NA} > X_{RA}$ if $\theta < b/2$ and $\beta < b/2$

(11) $X_{NN} > X_{RA}$ if $\theta < b/2$ and $\beta < b/2$

(12) $\pi_{RA} > \pi_{NA} > \pi_{RN} > \pi_{NN}$ if $\beta \neq 0$