Cash-Flow Business Taxation Revisited: Bankruptcy, Risk Aversion and Asymmetric Information*

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Abstract

It is well-known that cash-flow business taxes with full loss-offset, and their present-value equivalents, are neutral with respect to firms’ investment decisions when firms are risk-neutral and there are no distortions. We study the effects of cash-flow business taxation when there is bankruptcy risk, when firms are risk-averse, and when financial intermediaries face asymmetric information problems in financing heterogeneous firms. In these circumstances, investment decisions are distorted, with investment being less than in the full-information case. Cash-flow taxation corrects the distortion by inducing more investment in rent-generating projects and increasing social welfare. An ACE tax is equivalent to a cash-flow tax but is easier to implement under asymmetric information.

Key words: cash-flow tax, risk-averse firms, asymmetric information.

JEL Classification: H21, H25.
1 Introduction

A classic result in the design of business taxes due to Brown (1948) concerns the neutrality of cash-flow taxation. Investment decisions undertaken in a world of full certainty will be unaffected by a tax imposed on firms’ cash flows, assuming there is full loss-offsetting (and the tax rate is constant, as shown by Sandmo, 1979). In effect, a cash-flow tax will divert a share of the pure profits or rents from the firm’s owners to the government. This is of obvious policy interest since it represents a non-distorting source of tax revenue.

Not surprisingly, the Brown tax inspired a sizable literature on neutral business tax design, much of which generalized Brown’s neutrality result to taxes that are equivalent to cash-flow taxes in present value terms. Boadway and Bruce (1984) show that the cash-flow tax is a special case of a more general class of neutral business taxes that have the property that the present value of deductions for future capital costs (interest plus depreciation) arising from any investment just equals initial investment expenditures. (Current costs are assumed to be fully deductible on a cash basis when incurred, though they can be capitalized as well with no difficulty.) They characterized a general neutral business tax satisfying this property as follows. Investment expenditures are added to a capital account each year, and each year the capital account is depreciated at a rate specified for tax purposes. Capital costs deducted from the tax base in each tax year consist of the cost of capital plus the depreciation rate times the book value of the capital account. We refer to this as a Capital Account Allowance (CAA) tax. The CAA tax is neutral regardless of the depreciation rate used, as long as full loss-offsetting applies. Moreover, the depreciation rate used for tax purposes can be arbitrary and can vary from year to year. Its pattern can be chosen so that negative tax liabilities are mitigated. The firm itself can choose the depreciation rate, possibly contingent on minimizing tax losses in any given year. In effect, the CAA tax allows the firm to carry forward unused deductions for investment at the interest rate.

More generally, neutrality can be achieved by a business tax in which the present value of future tax bases just equals the present value of cash flows. An example of a cash-flow equivalent tax system of this sort is the Resource Rent Tax (RRT) proposed by Garnaut and Clunies-Ross (1975) for the taxation of non-renewable natural resources. In their version, firms starting out are allowed to accumulate negative cash flows in an account that rises each year with the cost of capital. Once the account becomes positive, cash flows are taxed as they occur. Like the Brown tax or the CAA tax, the RRT is neutral with respect to decisions by the firm, including extraction in the case of resource firms. Negative cash flows are carried forward at the interest rate to achieve the equivalent of cash-flow taxation.

These basic results continue to apply if returns to investment are uncertain, provided
firms are risk-neutral. Fane (1987) shows that neutrality holds under uncertainty as long as tax credits and liabilities are carried forward at the risk-free nominal interest rate, provided that tax credits and liabilities are eventually redeemed. Bond and Devereux (1995) show that the CAA tax remains neutral in the presence of uncertainty and the possibility of bankruptcy provided that a risk-free interest rate applied to the value of the capital account is used for the cost of capital deduction, that any unused negative tax credits are refunded in the event of bankruptcy, and that the valuation of risky assets satisfy the value additivity principle.\footnote{The value additivity principle implies that the present value of the sum of stochastic future payoffs is equal to the sum of the present values of these payoffs, and is consistent with a no-arbitrage principle in the valuation of assets.} The use of a risk-free discount rate reflects the assumption that there is no risk to the firm associated with postponing capital deductions into the future (i.e., no political risk). Boadway and Keen (2015) show that the same neutrality result applies to the RRT in the presence of uncertainty.

The above results focussed on taxes applied to real cash flows or their equivalent, what Meade (1978) called R-base cash-flow taxation. Bond and Devereux (2003) show that neutrality can be achieved using the more general case of cash-flow taxation proposed by Meade, referred to as (R+F)-base cash-flow taxation, in which both real and financial cash flows are included in the base. They also study the neutrality of the Allowance for Corporate Equity (ACE) tax, which is a version of the CAA tax that allows actual interest deductions alongside a cost of capital deduction for equity-financed investment. (They assume that there are no rents earned by bond-holders.) Notably, Bond and Devereux assume full information in the sense that banks can observe incomes of firms that claim to be bankrupt. As well, they do not allow firms to choose the quantity of investment. Panteghini (2006) examines the neutrality of the corporate tax under default risk. He shows that neutrality will generally be violated when shareholders can choose the timing of debt default, that is, when debt is unprotected. Recently, these results have been extended to consider the effect of cash-flow taxation on the entry decision of firms/entrepreneurs. Kanniainen and Panteghini (2012) show, using an option-value model for determining entry (and exit) of entrepreneurs, that cash-flow taxation distorts the entry decision unless the cash-flow tax rate is same as the wage tax rate potential entrepreneurs face in alternative employment.

These neutrality results have inspired various well-known policy proposals, some of which have been implemented. A cash-flow business tax was recommended by the US Treasury (1977), Meade (1978) in the UK, and the President’s Panel (2005) in the USA. The latter two both recommended additional cash-flow taxation to apply to financial institutions. At the same time, refundability of outstanding tax losses on firms that wind up was not
proposed. The Australian Treasury (2010) (the Henry Report) recommended an RRT for the mining industries in Australia. Several bodies have recommended an ACE corporate tax system, including the Institute for Fiscal Studies (1991), the Mirrlees Review (2011) and Institut d’Economia de Barcelona (2013). ACE taxes have been deployed in a few countries, including Brazil, Italy, Croatia and Belgium. Reviews of their use may be found in Klemm (2007), de Mooij (2011), Panteghini, Parisi, and Pighetti (2012), and Princen (2012). Cash-flow-type taxes with full loss-offset are used in the Norwegian offshore petroleum industry reviewed in Lund (2014), and the RRT was applied temporarily in the Australian mining industry.

The neutrality of cash-flow taxation no longer applies when the simple assumptions are relaxed. Suppose firms’ owners are risk-averse so that part of the return to investment is compensation for risk. A cash-flow tax applies to both rents and returns to risk-taking, and these two streams cannot be distinguished. As Domar and Musgrave (1944) famously show, risk-averse savers faced with a proportional tax on capital income with full loss-offset would be expected to increase the proportion of their portfolio held as risky assets, although the results become murkier when the proceeds from the tax are returned to savers by the government, as thoroughly discussed in Atkinson and Stiglitz (1980) and Buchholz and Konrad (2014). These results readily extend to an entrepreneurial firm, as shown in Mintz (1981). This non-neutrality is not necessarily a bad thing if the government is better able to diversify risk that private savers.

Additional problems arise if there are credit-market imperfections due to asymmetric information. This has been approached as a problem of adverse selection, first studied by Stiglitz and Weiss (1981) and subsequently by de Meza and Webb (1987). The latter authors show that if banks cannot observe the probability of a firm going bankrupt, there will be excessive credit extended to entrepreneurs. These results have been generalized by Boadway and Keen (2006) and Boadway and Sato (2011) to allow for ex post monitoring by banks to verify that firms claiming bankruptcy are truly insolvent. Moral hazard in credit markets can also give rise to market failure, as in Townsend (1979), Diamond (1984), Williamson (1986, 1987) and Bernanke, Gertler and Gilchrist (1996), where the costs of observing returns realized ex post by the lender generate an agency cost that tends to distort the market equilibrium. Parts of the literature on credit market failures resulting from asymmetric information are surveyed in Boadway and Tremblay (2005).

Finally, our analysis is also related to the literature that examines how conflicts between shareholders and bondholders affect the financial and investment decisions of firms. Relevant contributions in this literature include Leland (1998) and Childs, Mauer and Ott (2005) who examine the optimal capital structure of firms in the presence of shareholder-
bondholder conflicts, and characterizes how financial flexibility affects leverage, risk and debt maturity. The optimal capital structure is partly determined by the tax benefits of debt, and Childs, Mauer and Ott (2005) show that an increase in the corporate tax rate will tend to increase the firms leverage and reduce debt maturity.

In this paper, we revisit the use of cash-flow taxation as a rent-collecting device when firms might be risk-averse and asymmetries of information exist in capital markets. We do so in a simple partial equilibrium model of risk-averse entrepreneurs who vary in their productivity, so that returns to inframarginal entrepreneurs generate rents. We assume that banks can observe entrepreneurs’ types so there is no adverse selection. Investment outcomes are uncertain and entrepreneurs face the possibility of bankruptcy, which banks can verify only by engaging in costly monitoring. In these circumstances, investment is inefficiently low compared with the full-information case.

We study the effects of cash-flow taxation on both the entry decision of potential entrepreneurs (the extensive margin) and on the decision as to how much to borrow and invest (the intensive margin). The effect of the tax on intensive-margin decisions turns out to be especially important. Unlike in the literature discussed above, cash-flow taxation is not neutral. On the contrary, it tends to encourage rent-generating investment thereby acting as a corrective device in a distorted environment. To facilitate interpretation, we begin with a base case where entrepreneurs are risk-neutral. We then extend the analysis to allow for risk-averse entrepreneurs. Conducting the analysis using entrepreneurial firms is for simplicity. The analysis could be readily extended to public corporations. In our base case, we consider an R-base cash-flow tax that applies both to entrepreneurs and banks so banks can deduct monitoring costs. We then consider two alternative cases, one where the R-base tax does not apply to banks and the other where an (R+F)-base tax applies, which is equivalent to an ACE tax.

Some of our main results are as follows. With risk-neutral entrepreneurs and the possibility of bankruptcy, the R-base cash-flow tax is neutral in the absence of asymmetric information as in Bond and Devereux (1995, 2003), but distorts investment decisions if banks must incur monitoring costs when firms declare bankruptcy. Surprisingly, if banks can deduct monitoring costs from taxable income on bankrupt projects, the cash-flow tax increases investment while leaving bankruptcy risk and firms’ expected profits unchanged. If banks cannot deduct monitoring costs against the taxable income of the bankrupt projects they take over, which may be the more realistic case, the cash-flow tax no longer necessarily increases investment. Expected rents, government expected revenues and social surplus all increase with the tax rate when monitoring costs are deductible. In effect, the cash flow tax serves as a corrective device for the market failure arising from asymmetric information.
With risk-averse entrepreneurs, the cash-flow tax results in some risk-sharing between firms and the government. If there are no monitoring costs, or if such costs are tax deductible for banks, the cash-flow tax induces higher investment but is neutral with respect to bankruptcy risk and expected utility as in Domar and Musgrave (1944). An ACE tax achieves the same outcome as a cash-flow tax with monitoring costs deductible.

We begin by outlining the main elements of the model under risk-neutrality. We then study the effects of cash-flow taxation in the basic model, and consider some extensions including the ACE tax. Finally, we assume firms are risk-averse and analyze the effects of cash-flow taxation.

2 The Base Case with Risk-Neutral Entrepreneurs

2.1 Outline of the model

Our model is designed to reflect the following key features. Firms’ investments are heterogeneous so that infra-marginal investments generate pure profits or rents. The purpose of profit taxation is to tax these rents, and in a first-best world cash-flow profits taxation would do so in a non-distorting way. Our model departs from the first best by assuming that firms, who rely on the banks to finance their investments, face the possibility of bankruptcy, but that banks cannot observe without cost the profits of firms that declare bankruptcy. Banks can learn these profits at a cost by ex post monitoring or verification. The consequence is that relative to the full-information setting, there are too few loans. In this context, cash-flow taxation can actually encourage lending and improve social efficiency. The manner in which cash-flow taxation affects efficiency depends on whether the owners of firms, which we refer to as entrepreneurs, are risk-neutral or risk-averse.

In our model, there is a population of potential entrepreneurs with an identical endowment of wealth who can undertake an investment project. They differ in the productivity of their projects. We simplify our analysis by assuming that there is a single period so we can suppress the entrepreneurs’ consumption-savings decision and focus on production decisions. At the beginning of the period, potential entrepreneurs decide whether to enter a risky industry and invest their wealth in a risky project. Those who do not enter invest their wealth in a risk-free asset and consume the proceeds at the end of the period. Entrepreneurs who enter the risky industry choose how much to borrow to leverage their own equity investment, which determines their capital stock. After investment has been undertaken, risk is resolved. Those with good outcomes earn profits for the entrepreneur. Those with bad outcomes go bankrupt. Their production goes to their creditors, which are
risk-neutral competitive banks.

There are thus two decisions made by potential entrepreneurs. First, they decide whether to enter, which we can think of as an extensive-margin decision, and second, they decide how much to borrow to expand their capital, which is an intensive-margin decision. For simplicity, we suppress their labor income: all income comes from profits they earn if they enter the risky sector, or from their initial wealth if they do not. Adding labor income (as in Kanniainen and Panteghini (2012)) would make no substantial difference for our result on business taxation.

We assume that banks know the productivity of entrepreneurs, so can offer type-specific interest rates. The interest rate offered to a given type of entrepreneur depends on the borrowing the entrepreneur chooses. More borrowing increases the risk of bankruptcy, which in turn affects the expected profit of the lending bank. Since banks are competitive, their expected profits from loans to each type of entrepreneur will be zero in equilibrium, and this zero-profit condition determines the interest rate. Entrepreneurs know how their borrowing affects their interest rate, and that influences how much they borrow.

The details of the model with risk-neutral entrepreneurs are as follows. Later we consider the consequences of entrepreneurs being risk-averse and unable to insure against the risk of their uncertain incomes.

### 2.2 Details of the model with risk-neutral entrepreneurs

A continuum of potential entrepreneurs are all endowed with initial wealth $E$. For simplicity, we assume that the production function is linear in capital $K$. The average product of capital, denoted $R$, is constant, but differs across entrepreneurs, and is distributed over $[0, \bar{R}]$ by the distribution function $H(R)$. The value of output is subject to idiosyncratic risk, and the stochastic value of a type−$R$ entrepreneur’s output is $\tilde{\varepsilon} RK$, where $\tilde{\varepsilon}$ is distributed uniformly over $[0, \varepsilon_{\text{max}}]$, with density $g = 1/\varepsilon_{\text{max}}$. The expected value of $\varepsilon$ is:

\[
\bar{\varepsilon} \equiv \mathbb{E}[\tilde{\varepsilon}] = \frac{\varepsilon_{\text{max}}}{2} = \frac{1}{2g}
\]

We assume that the distribution of $\tilde{\varepsilon}$ is the same for all entrepreneurs, so they differ only by their productivity $R$. Capital is financed by the entrepreneur’s own equity and debt, and depreciates at the proportional rate $\delta$ per period. Entrepreneurs who do not enter invest all their wealth in a risk-free asset with rate of return $\rho$, so consume $(1+\rho)E$. Since all potential entrepreneurs have the same alternative income, those with the highest productivity as entrepreneurs will enter the entrepreneurial sector. Let $\bar{R}$ denote the productivity of the marginal entrepreneur.
Entrepreneurs who enter invest all their wealth in the risky firm. Then, $E$ will be the common value of own-equity of all entrepreneurs. The type−$R$ entrepreneur who has entered borrows an amount $B(R)$ so his aggregate capital stock is $K(R) = E + B(R)$. Let $B(R) \equiv \phi(R)K(R)$, where $\phi(R)$ is the leverage rate. Then $K(R)$ can be written:

$$K(R) = \frac{E}{1 - \phi(R)}$$  \hspace{1cm} (2)

We assume that there is a maximum value of the capital stock, such that $K(R) \leq \bar{K}$, and moreover that $E < \bar{K}$ so the entrepreneur’s wealth is less than the maximum size of the capital stock. By (2), this implies that $0 \leq \phi(R) \leq 1 - \frac{E}{\bar{K}} < 1$. Since we assume that all the entrepreneur's wealth is invested, the minimum level of capital for entrepreneurs who enter is $E$. Allowing entrepreneurs to invest only part of their wealth would complicate the analysis slightly without adding any insight. The entrepreneur’s capital stock is therefore in the range $K(R) \in [E, \bar{K}]$. The assumption of a maximal capital stock reflects the notion that after some point additional capital is non-productive. It is like a strong concavity assumption on the production function, which precludes extreme outcomes that would otherwise occur with linear production. In most of our analysis, we assume that entrepreneurs choose an interior solution so $K(R) \leq \bar{K}$ is not binding.

Since we assume that banks can identify entrepreneurs by type and set a type-specific interest rate, equilibrium analysis applies separately to entrepreneurs of each type.\(^3\) Accordingly, consider a representative type−$R$ entrepreneur and drop the identifier $R$ from all functions for simplicity. After the shock $\bar{\varepsilon}$ is revealed, an entrepreneur’s ex post after-tax profits (or return to own-equity) evaluated at the end of the period is given by:

$$\bar{\Pi} = \bar{\varepsilon}RK + (1 - \delta)K - (1 + r)B - \bar{T}$$  \hspace{1cm} (3)

where $\bar{T}$ is the tax paid and $r$ is the interest rate, so $(1 + r)B$ is the repayment of interest and principal on the borrowing $B$. The term $(1 - \delta)K$ is the value of capital remaining after production, given the depreciation rate $\delta$. The type-specific interest rate $r$ will depend upon the leverage $\phi$ chosen by the entrepreneur since this affects bankruptcy risk. The manner in which $\phi$ affects $r$ depends upon the behavior of the lending banks as discussed below.

\(^2\)If entrepreneurs were to invest part of their wealth in the safe asset, leverage would increase for any given level of investment. That would increase bankruptcy risk and the interest rate faced by the entrepreneur. If entrepreneurs face unlimited liability in the case of bankruptcy, there would be no incentive to invest less than total wealth in the risky project since the interest rate on borrowing will be higher than the rate of return on the safe asset. If there is limited liability in the case of bankruptcy, entrepreneurs may choose to hold wealth in the safe asset although that would result in higher interest costs on borrowing.

\(^3\)If wealth differed among entrepreneurs, leverage $\phi$ and therefore the interest rate could vary with both $R$ and $E$. This would not affect the qualitative results of our analysis.
We consider two forms of taxation: cash-flow and ACE taxation. In the absence of
market failures, these taxes are both non-distorting. However, as we shall see, in our asym-
metric information context both affect investment choices but in different ways. Moreover,
both serve partly to correct the market distortions. We focus on cash-flow taxation here,
and return to the ACE system later. Tax liability under cash-flow taxation, again evaluated
at the end of the period after $\tilde{\varepsilon}$ is revealed, is given by:

$$\tilde{T}^{CF} = \tau(\tilde{\varepsilon}RK - (1 + \rho)K + (1 - \delta)K) = \tau(\tilde{\varepsilon}RK - \rho K - \delta K) \quad (4)$$

where $\tau$ is the tax rate and, as noted above, $\rho$ is the risk-free interest rate. The tax liability
$\tilde{T}^{CF}$ is incurred by the firm as long as it is not bankrupt. If the firm goes bankrupt, the
bank pays taxes on the bankrupt cash flows. In addition, we allow for the possibility that
the bank gets to deduct costs of monitoring the firm for bankruptcy. The cash-flow tax base
in the middle expression consists of three terms. The first is the revenue of the firm, $\tilde{\varepsilon}RK$.
The second, $(1 + \rho)K$, is the end-of-period value of the deduction for investment. Since
investment $K$ occurs at the beginning of the period, we assume that the tax savings from
deducting investment are either refunded immediately or are carried over to the end of the
period with interest at rate $\rho$. Finally, the cash-flow tax is levied on selling or winding-up
the depreciated value of business assets, $(1 - \delta)K$, at the end of the period. Eq. (4) applies
whether $\tilde{T}$ is positive or negative, so implicitly assumes that the tax system allows full
loss-offsetting.

Using (3) and (4), ex post after-tax profits under cash-flow taxation may be written:

$$\bar{\Pi} = (1 - \tau)(\tilde{\varepsilon}RK + (1 - \delta)K) - (1 + r)B + \tau(1 + \rho)K \quad (5)$$

Entrepreneurs are confronted with bankruptcy when $\tilde{\varepsilon}$ is too low to meet debt repayment
obligations, that is, when $\bar{\Pi} < 0$. This occurs for entrepreneurs with $\tilde{\varepsilon} < \hat{\varepsilon}$, where $\hat{\varepsilon}$ (which
is specific to type $R$) satisfies:

$$0 = (1 - \tau)(\hat{\varepsilon}RK + (1 - \delta)K) - (1 + r)B + \tau(1 + \rho)K \quad (6)$$

In what follows, we refer to $\hat{\varepsilon}$ as bankruptcy risk. The higher the value of $\hat{\varepsilon}$, the greater the
chances of the entrepreneur going bankrupt. In the event of bankruptcy, the loan is not
repaid, and the remaining after-tax profits $(1 - \tau)(\hat{\varepsilon}RK + (1 - \delta)K) + \tau(1 + \rho)K$ go to the
lending bank. Combining (5) and (6), we obtain:

$$\bar{\Pi} = (1 - \tau)(\tilde{\varepsilon} - \hat{\varepsilon})RK \quad \text{for} \quad \tilde{\varepsilon} \geq \hat{\varepsilon} \quad (7)$$

Eq. (6) can be written, using $\phi = B/K$, as:

$$(1 - \tau)(\hat{\varepsilon}R + (1 - \delta)) - (1 + r)\phi + \tau(1 + \rho) = 0 \quad (8)$$
As this expression indicates, bankruptcy risk \( \varepsilon \) depends on both the leverage chosen by the entrepreneur, \( \phi \), and the interest rate, \( r \). The latter is determined by a competitive banking sector as follows. Assume that banks are risk-neutral and can observe \( R \) and \( \phi \) for each entrepreneur, but cannot observe \( \varepsilon \) or ex post profits. Thus, there is no adverse selection since banks know entrepreneurs’ types, but there is moral hazard since entrepreneurs may have an incentive to declare bankruptcy to avoid repaying the loan. Imperfection of the financial market due to asymmetric information is addressed by an ex post verification or monitoring cost in the event a firm declares bankruptcy. Following the financial accelerator model of Bernanke et al (1999), we assume that the verification cost is proportional to ex post output so takes the form \( c\varepsilon RK \), for \( \varepsilon \leq \hat{\varepsilon} \). This might reflect the fact that the verification cost includes the costs of seizing the firm’s output in a default.\(^4\) We assume for simplicity that there are no errors of monitoring. Then, only entrepreneurs with \( \bar{\varepsilon} < \hat{\varepsilon} \) will declare bankruptcy in equilibrium. The expected total monitoring cost for a given type of entrepreneur will be:

\[
\int_0^{\hat{\varepsilon}} c\bar{\varepsilon} RK \, d\bar{\varepsilon} = cRKg\frac{\bar{\varepsilon}^2}{2}
\]

so the expected monitoring cost increases with bankruptcy risk, \( \hat{\varepsilon} \). This specific form of the monitoring cost is not critical to our results. It is chosen for analytical convenience.

In the event of bankruptcy, the firm no longer pays \((1 + r)B\), and its profits go to the bank. Using (5) these profits become:

\[
\bar{\Pi} = (1 - \tau)\left(\bar{\varepsilon}RK + (1 - \delta)K\right) + \tau(1 + \rho)K \quad \text{for} \quad \bar{\varepsilon} < \hat{\varepsilon} \quad \text{(10)}
\]

As this expression indicates, we assume that negative tax liabilities owing to bankrupt firms are refundable to the banks. Competition among banks ensures that expected profits earned from lending to the representative entrepreneur of each type are zero. We assume that banks will not go bankrupt, so they pay the risk-free interest rate \( \rho \) on their deposits, and also that they incur no costs of operation for simplicity.

We assume in our base-case analysis that banks can deduct monitoring costs from the taxable income of bankrupt projects and that these tax deductions are refundable. Later we consider the consequences of not letting them be deductible. With monitoring costs deductible, zero-expected bank profits imply the following, using the fact that debt is only

\(^4\)Bernanke and Gertler (1989) introduced a fixed verification cost in a business cycle model where there is asymmetric information between lenders and borrowers about the realized return on risky projects, while Townsend (1979) explored the design of debt contracts with verification costs that could either be fixed or functions of realized project output. See also Bernanke et al (1996) for an analysis of the implications of agency costs in lending contracts arising from asymmetric information about project outcome.
repaid if $\tilde{\varepsilon} > \hat{\varepsilon}$,
\[
(1 + \rho)B = (1 + r)B \int_{\tilde{\varepsilon}}^{\varepsilon_{max}} g d\tilde{\varepsilon} + \int_{0}^{\hat{\varepsilon}} \tilde{\Pi} g d\tilde{\varepsilon} - (1 - \tau) c R K g \tilde{\varepsilon}^2 / 2
\]

Substituting (10) for $\tilde{\Pi}$ and integrating, we obtain:
\[
(1 + \rho)B = (1 + r)B \int_{\tilde{\varepsilon}}^{\varepsilon_{max}} g d\tilde{\varepsilon} + \int_{0}^{\hat{\varepsilon}} (1 - \tau) \left( \tilde{\varepsilon} R K + (1 - \delta) K \right) \tau (1 + \rho) K g d\tilde{\varepsilon} - (1 - \tau) c R K g \tilde{\varepsilon}^2 / 2
\]
\[
= (1 + r) B (1 - g \tilde{\varepsilon}) + (1 - \tau) R K g \tilde{\varepsilon}^2 / 2 + (1 - \tau) (1 - \delta) K g \tilde{\varepsilon} - (1 - \tau) c R K g \tilde{\varepsilon}^2 / 2
\]

Eq. (11) reflects the fact that for $\varepsilon \leq \tilde{\varepsilon}$, the banks retain the bankrupt firm’s after-tax profits, but face tax-deductible verification costs $c \varepsilon R K$. This zero-profit condition, which applies for each type of entrepreneur, determines the interest rate the entrepreneur of a given type pays, given their bankruptcy risk, or equivalently, their leverage.

Our assumed tax treatment of banks deserves explanation. The Meade Report (1978) proposed two alternative forms of cash-flow taxation. R-based cash-flow tax would include only real cash-flows, whereas R+F-based cash-flow taxation would include both real and financial cash-flows. Our model assumes the former since banks pay no cash-flow tax on their financial profits. The absence of cash-flow taxation on banks’ financial cash-flows is innocuous given our assumption that banks earn no pure profits or rents and are risk-neutral. Expected cash flows, and therefore expected tax liabilities, would be zero. On the other hand, financial institutions may or may not be liable for R-based cash-flow taxation. The argument for excluding them is that it is difficult to distinguish between their real and financial transactions. Our base case in which monitoring costs are deductible corresponds with the case where the R-based cash-flow tax applies to banks. It serves as a useful benchmark. Later, we consider the case where monitoring costs are not deductible, which would be the case when financial institutions are exempt from the R-based cash-flow tax.

The bankruptcy condition (6) and the bank’s zero-profit condition (11) jointly determine the relations among $r$, $\phi$ and $\hat{\varepsilon}$. By combining (6) and (11), we can eliminate $(1 + r)B$ and obtain a relationship between $\phi$ and $\hat{\varepsilon}$ as shown in the following lemma.

**Lemma 1** The leverage rate $\phi \equiv B / K$, for $0 < \phi < 1 - E / K$, is given by:
\[
\phi(\hat{\varepsilon}, R, \tau, c) = \frac{1 - \tau}{1 + \rho} \left( (1 - g \hat{\varepsilon}) - \frac{c g \hat{\varepsilon}}{2} \right) \hat{\varepsilon} R + (1 - \delta) + \tau
\]

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The proofs of all lemmas are given in the Appendix. Routine differentiation of (12) gives properties of \( \phi(\hat{\varepsilon}, R, \tau, c) \) that are useful in what follows:

\[
\phi_{\hat{\varepsilon}} = \frac{1 - \tau}{1 + \rho} (1 - g\hat{\varepsilon} - cg\hat{\varepsilon})R; \quad \phi_c = -\frac{1 - \tau}{1 + \rho} \frac{Rg\hat{\varepsilon}^2}{2}; \quad \phi_R = \frac{1 - \tau}{1 + \rho} \left( 1 - \frac{g\hat{\varepsilon}}{2} - \frac{cg\hat{\varepsilon}}{2} \right);
\]

\[
\phi_{\tau} = \frac{1 - \phi}{1 - \tau}; \quad \phi_{\hat{\varepsilon}c} = -\frac{1 - \tau}{1 + \rho} (1 - c) Rg\hat{\varepsilon}; \quad \phi_{\hat{\varepsilon}R} = -\frac{1 - g\hat{\varepsilon} - cg\hat{\varepsilon}}{1 + \rho} R
\]

Entrepreneurs understand how the leverage they choose affects the probability of bankruptcy and the interest rate they face through the bankruptcy condition (6) and the zero-profit condition (11). Therefore, they know the relationship between \( \phi \) and \( \hat{\varepsilon} \) in (12), and how it takes account of the interest rate they face. In what follows, we take advantage of Lemma 1 to suppress the interest rate \( r \) from our analysis. While in practice, entrepreneurs choose leverage \( \phi \), it is convenient for us to assume in our analysis that they choose bankruptcy risk \( \hat{\varepsilon} \), which is related to leverage via (12). We proceed by deriving an expression for the entrepreneur’s expected profits as a function of \( \hat{\varepsilon} \).

Prior to \( \hat{\varepsilon} \) being revealed, the expected profits of a representative entrepreneur of a given type are \( \Pi \equiv \int_{\hat{\varepsilon}}^{\xi_{\text{max}}} \Pi (\hat{\varepsilon}) g d\hat{\varepsilon} \). (Recall that for \( \hat{\varepsilon} < \xi \), profits are claimed by the bank.) Given the expression for \( \Pi \) in (5), this becomes:

\[
\Pi = \int_{\hat{\varepsilon}}^{\xi_{\text{max}}} \left( (1 - \tau) (\xi R K + (1 - \delta) K) - (1 + r) B + \tau (1 + \rho) K \right) g d\hat{\varepsilon}
\]

Using (11) to eliminate \((1 + r)B(1 - g\hat{\varepsilon})\) from (14), and using (1) and (2) along with \( B = K - E \), (14) may be written:

\[
\Pi = \left( \frac{1 - \tau}{1 - \phi(\hat{\varepsilon}, R, \tau, c)} \left( \xi R - \delta - \rho - cRg\hat{\varepsilon}^2 \right) + 1 + \rho \right) E \equiv \pi(\hat{\varepsilon}, R, \tau, c) E
\]

where \( \pi(\hat{\varepsilon}, R, \tau, c) \) is expected profit per unit of own equity. For future use, differentiate \( \pi(\cdot) \) in (15) with respect to \( \hat{\varepsilon} \) to obtain:

\[
\pi_{\hat{\varepsilon}} = \frac{1 - \tau}{1 - \phi} \left( \Delta(\hat{\varepsilon}, R, \tau, c) \left( \xi R - \delta - \rho - \frac{cRg\hat{\varepsilon}^2}{2} \right) - c\hat{\varepsilon}gR \right)
\]

where

\[
\Delta(\hat{\varepsilon}, R, \tau, c) \equiv \frac{\phi(\hat{\varepsilon}, R, \tau, c)}{1 - \phi(\hat{\varepsilon}, R, \tau, c)}
\]

Using (12), \( \pi(\hat{\varepsilon}, R, \tau, c) \) satisfies the following lemma.

---

We could instead have used (12) to determine \( \hat{\varepsilon} \) as a function of \( \phi \), and obtained derivatives of \( \hat{\varepsilon} \) with respect to \( \phi \) and the other variables. We could then use \( \phi \) as the choice variable of entrepreneurs. While this would more accurately reflect entrepreneurial choices, it would make the analysis more complicated and would not change the results.
Lemma 2

\[ \pi(\hat{\varepsilon}, R, \tau, c) = \frac{1 - \tau}{1 - \phi(\cdot)} \frac{Rg}{2} (\varepsilon_{\text{max}} - \hat{\varepsilon})^2 \]

Therefore, expected profits can be written:

\[ \Pi = \pi(\hat{\varepsilon}, R, \tau, c)E = \frac{1 - \tau}{1 - \phi(\cdot)} \frac{Rg}{2} (\varepsilon_{\text{max}} - \hat{\varepsilon})^2 E \] (18)

3 Behavior of Risk-Neutral Entrepreneurs

Recall that entrepreneurs make two choices in sequence. First, they decide whether to undertake risky investments, given their productivity \( R \). This is the extensive-margin decision. Then, if they enter, they decide how much to borrow to acquire more capital over and above their own equity, \( E \). This is their intensive-margin decision. Once their shock \( \hat{\varepsilon} \) is revealed, their after-tax profits and therefore ex post utility are determined. We consider the intensive and extensive decisions in reverse order for a representative entrepreneur of a given type, and continue to suppress the type identifier \( R \) for simplicity. Since we assume risk-neutrality in this basic model, the expected utility of entrepreneurs, and therefore their objective function, is given by \( \Pi = \pi(\hat{\varepsilon}, R, \tau, c)E \) in (15) or (18).

3.1 Choice of leverage: intensive margin

As mentioned, given (12) the choice of leverage \( \phi \), which determines \( K = E/(1 - \phi(\cdot)) \), is essentially the same as the choice of \( \hat{\varepsilon} \). This follows because, even though \( \phi \) is not necessarily monotonic in \( \hat{\varepsilon} \), \( \phi_{\hat{\varepsilon}} < 0 \) by (13). Differentiating (18) with respect to \( \hat{\varepsilon} \), we obtain:

\[ \frac{d\Pi}{d\hat{\varepsilon}} = \pi_{\hat{\varepsilon}}E = \left( \Delta(\hat{\varepsilon}, R, \tau, c) - \frac{2}{\varepsilon_{\text{max}} - \hat{\varepsilon}} \right) \frac{(1 - \tau)Rg}{1 - \phi} (\varepsilon_{\text{max}} - \hat{\varepsilon})^2 E \] (19)

where \( \Delta(\hat{\varepsilon}, R, \tau, c) \) is defined in (17).

Let \( \hat{\varepsilon}^* \) be the optimal choice of \( \hat{\varepsilon} \). It could be in the interior or it could take on corner solutions at the top or bottom. From (12), \( \hat{\varepsilon}^* \) takes on a minimum value of \( \hat{\varepsilon}^* = 0 \) when \( \phi \leq \phi(0, R, \tau, c) = (1 - \tau)(1 - \delta)/(1 + \rho) + \tau \). The maximum value of \( \hat{\varepsilon}^* \) satisfies \( \phi(\hat{\varepsilon}, R, \tau, c) = 1 - E/K \), which is assumed to be smaller than \( \varepsilon_{\text{max}} \) for any entrepreneur type. If it is in the interior, \( d\Pi/d\hat{\varepsilon} = 0 \), so

\[ \Delta(\hat{\varepsilon}^*, R, \tau, c) = \frac{2}{\varepsilon_{\text{max}} - \hat{\varepsilon}^*} > 0 \] (20)
To see how the level of $R$ affects leverage and bankruptcy risk, assume again that $\hat{\epsilon}^*$ is in the interior so $d\Pi/d\hat{\epsilon} = 0$ in (19). If the second-order conditions are satisfied, we have:

$$\frac{d^2\Pi}{d\hat{\epsilon}^2} \propto \Delta\hat{\epsilon}(\hat{\epsilon}, R, \tau, c) - \frac{2}{(\epsilon_{\text{max}} - \hat{\epsilon})^2} < 0$$

Rewrite elements of (13) as

$$\phi_{\hat{\epsilon}} = \frac{1 - \tau}{1 + \rho} \left(1 - g\hat{\epsilon} - cg\hat{\epsilon}\right)R, \quad \phi_R = \frac{1 - \tau}{1 + \rho} \left(1 - \frac{g\hat{\epsilon}}{2} - \frac{cg\hat{\epsilon}}{2}\right)\hat{\epsilon}$$

Since $\Delta(\hat{\epsilon}^*, R, \tau, c) > 0$ for $\hat{\epsilon}^*$ in the interior by (20), the following lemma is apparent.

**Lemma 3** For $\hat{\epsilon}^*$ in the interior, $\phi_{\hat{\epsilon}} > 0$, $\phi_R > 0$ and $\phi_{\hat{\epsilon}R} > 0$.

The following lemma then also applies.

**Lemma 4** Assuming that $\phi_{\hat{\epsilon}} > 0$ at $\hat{\epsilon}^*$ and the second-order conditions are satisfied,

$$\frac{d\hat{\epsilon}^*}{dR} > 0$$

Thus, the probability of bankruptcy increases with the productivity $R$ of the entrepreneur. This occurs because entrepreneurs with higher productivity choose higher leverage, by Lemma 3. Although a higher value of $R$ tends to reduce bankruptcy risk, the impact on bankruptcy risk is more than offset by the increase in leverage.

### 3.2 Decision to undertake risky investment: extensive margin

Ex ante, entrepreneurs decide whether to undertake the risky investment or to opt for the risk-free option. In the risk-free option, they invest their wealth $E$ at a risk-free return $\rho$, leading to consumption of $(1 + \rho)E$. They enter if their expected income as an entrepreneur, given by $\Pi$ in (15) or (18), is at least as great as their certain income if they invest their wealth in a safe asset and obtain consumption of $(1 + \rho)E$, that is,

$$\Pi = \bar{\pi}(\hat{\epsilon}, R, \tau, c)E \geq (1 + \rho)E \quad \text{or} \quad \bar{\pi}(\hat{\epsilon}, R, \tau, c) \geq 1 + \rho \quad (21)$$

Differentiating $\bar{\pi}(\cdot)$ in Lemma 2 by $R$ and using $\phi_R > 0$ by Lemma 3, we obtain that $\bar{\pi}(\cdot)$ is increasing in $R$. Given that $\hat{\epsilon}$ is being optimized, the cutoff value of $R$, denoted $\hat{R}$, will be uniquely determined by $\bar{\pi}(\hat{\epsilon}, \hat{R}, \tau, c) = 1 + \rho$. Using the expression for $\bar{\pi}$ in (15), the following lemma is apparent.
Lemma 5 The cutoff value of $\bar{R}$ is determined by:

$$\bar{R} - \delta - \rho - \frac{c\hat{R}g \hat{\varepsilon}^2}{2} = 0 \tag{22}$$

Entrepreneurs with $R > \bar{R}$ enter the risky sector and earn a rent. Those with $R < \bar{R}$ invest their wealth in a risk-free asset, so earn no rent. Note that for the marginal entrepreneur, Lemma 5 implies by (16) that $\partial \Pi / \partial \hat{\varepsilon} < 0$. Therefore, the marginal entrepreneur chooses $\hat{\varepsilon} = 0$ and incurs no bankruptcy risk. Given that $\hat{\varepsilon}^*$ is increasing in $R$ by Lemma 4, higher productivity entrepreneurs will face bankruptcy risk. This has implications for the effect of the cash-flow tax in what follows. To study this, consider first the social optimum as a benchmark.

4 The Social Optimum

To study the efficiency properties of cash-flow business taxation, it is useful to characterize the full-information social optimum, that is, the outcome where $c = 0$ so banks can observe without cost the output of the bankrupt firms. Social surplus includes only the surplus of projects of entrepreneurs who invest in the risky sector since no surplus is generated either by the banks, which earn zero expected profits, or by potential entrepreneurs who invest in the safe outcome and earn $(1 + \rho)E$. Expected social surplus can be defined as the expected value of production by entrepreneurs less the opportunity cost of financing the entrepreneurs’ capital. Financing costs include the cost of both debt and equity finance, so are given by $(1 + \rho)B + (1 + \rho)E = (1 + \rho)K$.

For the representative entrepreneur of type $-R$, end-of-period expected social surplus $S(R)$ can be written as follows, using $K = E/(1 - \phi)$:

$$S(R) = \int_{\hat{\varepsilon}}^{\hat{\varepsilon}_{\text{max}}} (\tilde{\varepsilon}RK + (1 - \delta)K) g d\tilde{\varepsilon} - (1 + \rho)K = \left(\bar{R} - \delta - \rho\right) E \frac{1}{1 - \phi(\cdot)} \tag{23}$$

where $\hat{\varepsilon}$ satisfies (12) with $c = 0$ and $\tau = 0$, or:

$$\phi(\hat{\varepsilon}, R, 0, 0) = \frac{1}{1 + \rho} \left(1 - \frac{g\hat{\varepsilon}}{2}\right) \hat{\varepsilon}R + (1 - \delta) \tag{24}$$

Note that $S(R)$ includes the surplus earned by the investments that go bankrupt since this accrues to the banks. This expression for $S(R)$ applies whether taxes are in place or not.

In a social optimum, both the extensive and intensive margins are optimized. Entry is optimized if $S(R) = 0$ for the marginal entrepreneur, or by (23),

$$\bar{R}^0 - \delta - \rho = 0 \tag{25}$$
where $\hat{R}^o$ is marginal entrepreneur in the social optimum. All entrepreneurs $R \geq \hat{R}^o$ enter in the social optimum. Changes in leverage, or equivalently in bankruptcy risk, affect $S(R)$ in (23) as follows:

$$\frac{dS(R)}{d\hat{\varepsilon}} = \frac{\phi_\varepsilon}{(1-\phi)^2} (\pi R - \delta - \rho) E$$

This implies by (25) that $\hat{R}^o$, $dS(\hat{R}^o)/d\hat{\varepsilon} = 0$ for the marginal entrepreneur so social surplus is independent of leverage and $K$. For $R > \hat{R}^o$, $dS(R)/d\hat{\varepsilon} > 0$ for all $\hat{\varepsilon}$ since $\phi_\varepsilon > 0$ by Lemma 3. Inframarginal entrepreneurs will therefore maximize leverage such that $K = \overline{K}$.

As expected, when $c = 0$ so the full-information social optimum is achieved, the cash-flow tax has no effect on market outcomes. It simply diverts rents from infra-marginal entrepreneurs to the government. To see this, consider first the extensive-margin decision. When $c = 0$, (22) implies that $\pi \hat{R} - \delta - \rho = 0$ so $R$ is independent of $\tau$. Thus, $\hat{R} = \hat{R}^o$ by (25) so entry is socially optimal. Next, consider the effect of the cash-flow tax on leverage. With $c = 0$, (16) implies:

$$\frac{d\Pi}{d\hat{\varepsilon}} = \pi \varepsilon E = \frac{1-\tau}{1-\phi} \Delta(\hat{\varepsilon}, R, \tau, c)(\pi R - \delta - \rho) E$$

For the marginal entrepreneur, (25) implies that $\pi_\varepsilon = 0$, so $d\Pi/d\hat{\varepsilon}|_{R=\hat{R}} = 0$. Therefore, leverage $\phi$ and thus $K$ are indeterminate for the marginal entrepreneur and independent of $\tau$. For inframarginal entrepreneurs, $\pi \hat{R} - \delta - \rho > 0$ since $R > \hat{R}$, so $\pi_\varepsilon$ has the same sign as $\Delta(\cdot) = \phi_\varepsilon/(1-\phi)$, which is positive by Lemma 3. Therefore, $\hat{\varepsilon}$ takes its maximum value with $\phi(\hat{\varepsilon}^*, R, \tau, c) = 1 - E/\overline{K}$. Since $\phi_\varepsilon > 0$ and $\phi_\tau > 0$ by (13), we have that $d\hat{\varepsilon}^*/d\tau < 0$ to keep $\phi$ constant. While $\hat{\varepsilon}^*$ changes, $\tau$ does not distort $\phi$ or the capital stock $K = \overline{K} = E/(1-\phi)$.

When banks must incur a monitoring cost $c$ to observe the profits of bankrupt firms, the social optimum will not be achieved. We saw above that Lemma 5 and (16) imply that $\hat{\varepsilon}^* = 0$ for the marginal entrepreneur. Therefore by (22), the productivity of the marginal entrepreneur satisfies $\pi \hat{R} - \delta - \rho = 0$, which implies that $\hat{R} = \hat{R}^o$ so entry is optimal. At the same time, since $d\hat{\varepsilon}/dR > 0$ by Lemma 4, $\hat{\varepsilon}^*$ is in the interior for large enough values of $R$, so leverage and therefore investment will be below the maximum level obtained in the social optimum. The implication is that while entry is optimal in the presence of imperfect information, there is too little investment for entrepreneurs that incur bankruptcy risk.

Before turning to the implications of cash-flow taxation in the imperfect information setting, it is useful to define constrained social surplus as social surplus less the costs of monitoring incurred by the banks since the government confronts the same information problem that the banks do. For a type-$R$ entrepreneur, constrained social surplus can be
expressed as follows, analogous to (23):

$$\mathcal{S}(R) = \left(\varepsilon R - \delta - \rho - c R g \frac{\varepsilon^2}{2}\right) \frac{E}{1 - \phi(\cdot)}$$  \hspace{1cm} (27)

Combining the entrepreneur’s expected profits in (14) with the bank’s zero profits expression (11), we obtain:

$$\Pi - (1 + \rho)E = \int_0^{\varepsilon_{\text{max}}} \left( (1 - \tau)(\varepsilon RK + (1 - \delta)K) - (1 + \rho)(B + E) \right) g d\varepsilon - (1 - \tau)cRKg \frac{\varepsilon^2}{2}$$

$$= (1 - \tau)\mathcal{S}(R)$$  \hspace{1cm} (28)

In the absence of taxation, maximizing private surplus $\Pi - (1 + \rho)E$ maximizes constrained social surplus, but that will no longer be the case with $\tau > 0$. We use (28) below to interpret the efficiency consequences of cash-flow taxation in an information-constrained setting.

To summarize, in the full-information social optimum, infra-marginal entrepreneurs maximize leverage and choose $K = \overline{K}$, while marginal entrepreneurs are indifferent to the level of $K$. The cash-flow tax has no effect on entry or leverage, but diverts to the government the rents of infra-marginal entrepreneurs. If there are monitoring costs, entry remains optimal and marginal entrepreneurs assume no bankruptcy risk, while some infra-marginal entrepreneurs under-invest.

## 5 Cash-Flow Taxation with Risk-Neutral Entrepreneurs

The model discussed in the previous sections includes both bankruptcy, when entrepreneurs are unable to repay their loans fully, and asymmetric information, in the sense that banks can only verify bankruptcy with costly ex post monitoring. In this section, we consider the effect of cash-flow taxation on entry and leverage as well as on after-tax profits, tax revenue and social surplus.

The leverage decision for an inframarginal type-$R$ entrepreneur is governed by (19), where $d\Pi/d\hat{\varepsilon} = 0$ if $\hat{\varepsilon}^*$ is in the interior. To determine the effect of taxes on leverage, differentiate $\phi(\hat{\varepsilon}(\cdot), R, \tau, c)$ to obtain:

$$\frac{d\phi}{d\tau} = \phi_\varepsilon \frac{d\hat{\varepsilon}^*}{d\tau} + \phi_\tau$$  \hspace{1cm} (29)

where $\phi_\tau = (1 - \phi)/(1 - \tau)$ by (13). To evaluate (29), we can use (20) to obtain the following lemma.

**Lemma 6** Assume $\hat{\varepsilon}^*$ is in the interior. Then,

$$\frac{d\hat{\varepsilon}^*}{d\tau} = 0$$
Thus, the cash-flow tax does not affect bankruptcy risk for firms with $\hat{\varepsilon}^*$ in the interior, although it does affect leverage $\phi$ by (13) and therefore $K$. Note that this result depends on the deductibility of monitoring costs from the taxable income of bankrupt firms. We return to this below. Using Lemma 6, (29) reduces to

$$\frac{d\phi}{d\tau} = \phi_{\tau} = \frac{1 - \phi}{1 - \tau} > 0$$  

(30)

While the tax does not affect bankruptcy risk, it does increase leverage and investment.

Some explanation for this comes from the following lemma.

**Lemma 7** For $\hat{\varepsilon}$ in the interior,

$$\frac{dr}{d\tau} < 0$$  

(31)

The intuition is that the cash flow tax allows the banks to claim a refund of the opportunity cost of investment, $\tau(1 + \rho)K$, on bankrupt projects. This increases the gain that the bank can collect from the bankrupt entrepreneurs, which improves its expected profits and thus leads to a reduction in $r$. By reducing $r$, the increase in $\tau$ induces entrepreneurs to borrow and therefore invest more.

Consider now the extensive margin decision. The cash-flow tax is neutral with respect to entry. This follows from the fact that the marginal entrepreneur $\hat{R}$ satisfies (22). Since $\hat{\varepsilon}$ independent of $\tau$, so is $\hat{R}$ and therefore entry.

Next, consider the effect of the tax on expected profits of the firm. Lemma 2 applies, so $\pi$ is given by:

$$\pi = \frac{1 - \tau}{1 - \phi} \frac{Rg}{2} (\varepsilon_{\text{max}} - \hat{\varepsilon})^2 \equiv D \frac{Rg}{2} (\varepsilon_{\text{max}} - \hat{\varepsilon})^2$$  

(32)

Using (30), $D \equiv (1 - \tau)/(1 - \phi)$ is independent of $\tau$, so expected profits are as well. Therefore, while the tax increases leverage and therefore $K$, it leaves expected after-tax profits unchanged. This is analogous to the Domar and Musgrave (1944) result albeit for a different reason in this context since entrepreneurs are risk-neutral. We obtain a similar result below for risk-averse entrepreneurs.

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$^6$Proof:

$$D_{\tau} = -\frac{1}{1 - \phi} + \frac{1 - \tau}{(1 - \phi)^2} \phi_{\tau} = -\frac{1}{1 - \phi} + \frac{1 - \tau}{(1 - \phi)^2} \frac{1 - \phi}{1 - \tau} = 0$$  

\[\blacksquare\]
Expected government revenue from the cash-flow tax can be written, using (4) and the tax deductibility of monitoring costs, as:

\[
T = \tau \int_{\bar{R}} (\bar{R} - \rho - \delta - cRg \hat{\varepsilon}^2/2) \frac{E}{1 - \phi} dH(R) \equiv \tau \bar{Y}
\]  

(33)

where \( \bar{Y} \) is the aggregate expected tax base and \( H(R) \) has been defined as the distribution of entrepreneur types. Given from above that neither \( \hat{R} \) nor \( \hat{\varepsilon} \) are affected by the tax, differentiating \( T \) yields:

\[
\frac{dT}{d\tau} = \bar{Y} + \tau \int_{\bar{R}} (\bar{R} - \rho - \delta - cRg \hat{\varepsilon}^2/2) \frac{E}{(1 - \phi)^2} \frac{d\phi}{d\tau} dH(R) > 0
\]  

(34)

where the inequality follows from (30). The first term is the mechanical effect of an increase in the tax rate on revenues which is positive. The second term is also positive given that leverage increases with the tax rate as shown above. Since leverage increases with \( \tau \), more rents are created by the additional investment and this induces an increase in \( \bar{Y} \).

Finally, consider the effect of the tax on constrained expected social surplus \( \bar{S} \). Using \( \bar{\Pi} - (1 + \rho)E = (1 - \tau)\bar{S} \) from (28), we have:

\[
\bar{S} = \frac{\bar{\Pi} - (1 + \rho)E}{1 - \tau}
\]

Since a tax increase leaves \( \bar{\Pi} = \pi E \) unchanged, it will increase \( \bar{S} \). In effect, the tax induces the firms to increase leverage while keeping \( \hat{\varepsilon} \) constant. To see how the tax affects \( K \), start by substituting \( D = (1 - \tau)/(1 - \phi) \) into (2) which gives \( K = ED/(1 - \tau) \). Let \( K_0 \) be the level of capital chosen by the entrepreneur in the absence of taxation, so \( K_0 = ED \) and \( K = K_0/(1 - \tau) \), implying:

\[
\Delta K = K - K_0 = \frac{K_0}{1 - \tau} - K_0 = \frac{\tau K_0}{1 - \tau} > 0
\]

Therefore, introducing the tax increases \( K \) and as can be seen from (27) \( \bar{S} \) increases.\(^7\) Equivalently, the increase in \( K \) holding \( \hat{\varepsilon} \) constant generates more pre-tax profits, or rents.

\(^7\)The marginal effect of the tax on capital can be obtain as follows: Differentiating (2), we obtain:

\[
\frac{dK}{d\phi} = \frac{E}{(1 - \phi)^2} = \frac{K}{1 - \phi}
\]

Differentiating \( D = (1 - \tau)/(1 - \phi) \), which is constant, gives

\[
\frac{d\phi}{d\tau} = \frac{1 - \phi}{1 - \tau}
\]

Combining these equations, we obtain:

\[
\frac{dK}{d\tau} = \frac{dK}{d\phi} \frac{d\phi}{d\tau} = \frac{K}{1 - \phi} \frac{1 - \phi}{1 - \tau} = \frac{K}{1 - \tau}
\]
The government taxes away those profits, leaving after-tax expected profits unchanged and improving constrained expected social surplus. Thus, while the no-tax outcome replicates the constrained social optimum, implementing a cash-flow tax improves social outcomes without changing firms’ expected profits. It does so by breaking the connection between leverage and bankruptcy risk. This reflects the fact that levels of $K$ in the absence of the tax are less than in the unconstrained social optimum for some entrepreneurs as discussed above.

The optimal tax rate for a given entrepreneur-type would be that which just induced the entrepreneur to choose maximum leverage. The government cannot observe $R$ so cannot implement optimal type-specific tax rates.

The main results of the analysis in the base-case model are summarized as follows.

**Proposition 1** With risk-neutral entrepreneurs, equilibrium has the following properties:

i. Entrepreneurs with average product $R$ above some threshold level $\hat{R}$ enter the risky industry and earn a rent. For those with $K$ in the interior, leverage $\phi$ and bankruptcy risk $\hat{\varepsilon}^*$ are increasing with $R$.

ii. In the absence of taxation, entry is socially efficient in equilibrium, but leverage and therefore investment are below the full information socially optimal levels.

iii. Assuming that the banks can deduct monitoring costs from the income tax on bankrupt projects, leverage increases with the tax rate, while bankruptcy risk and expected profits remain unchanged, and expected tax revenues increases. Expected rents and expected social surplus both increase, so the cash-flow tax act as a corrective device.

### 6 Alternative Tax Systems

The above analysis assumed that the government imposed a cash-flow tax with monitoring costs deductible from the taxable income received by the banks on bankrupt projects. While this is a useful benchmark case, the assumption that monitoring costs are deductible is a strong one unless the banks themselves are liable for $R$-based cash-flow taxation. As mentioned above, extending the $R$-based tax to financial institutions is challenging given the difficulty of distinguishing real from financial transactions. In this section we explore two realistic alternatives. In the first one, corresponding to the case where the $R$-based cash-flow tax does not apply to banks, we assume that monitoring costs are not deductible, although banks are still liable for taxes owing on bankrupt projects. In this case, the strong welfare-improving qualities of the previous analysis are weakened. The second case is the
ACE tax system, which applies to both real and financial transactions, including those of the banks. This case restores the properties of the basic model since banks can deduct monitoring costs from their taxable income. A third case we could have considered is the standard corporate tax that most countries currently deploy. Analysis of it in our model is relatively complex for the additional insight gained. We discuss the standard corporate tax briefly after dealing with the ACE tax to which it is most closely related.

6.1 Cash-flow taxation with monitoring costs non-deductible

Consider now the case where banks cannot deduct monitoring costs from taxable income of bankrupt firms. In this case, condition (11) for banks' zero expected profits becomes:

\[(1 + \rho)B = (1 + r)B(1 - g\hat{\varepsilon}) + (1 - \tau)RKg\frac{\hat{\varepsilon}^2}{2} + ((1 - \tau)(1 - \delta) + \tau(1 + \rho))Kg\hat{\varepsilon} - cRKg\frac{\hat{\varepsilon}^2}{2}\]  

(35)

and the leverage rate, derived in Lemma 1 for the case where monitoring costs are deductible, is now given by:

\[\phi(\hat{\varepsilon}, R, \tau, c) = \frac{1 - \tau}{1 + \rho}(1 - g\hat{\varepsilon})\hat{\varepsilon}R + \frac{1 - \tau}{1 + \rho}(1 - \delta) + \tau - cRg\frac{\hat{\varepsilon}^2}{2}\]  

(36)

Differentiation of (36) gives the analogue of (13):

\[\phi_{\varepsilon} = \frac{1 - \tau}{1 + \rho}(1 - g\hat{\varepsilon})\hat{\varepsilon}R; \quad \phi_c = -\frac{Rg}{2(1 + \rho)}\hat{\varepsilon}^2; \quad \phi_R = \frac{1 - \tau}{1 + \rho}(1 - g\hat{\varepsilon})\hat{\varepsilon}R - \frac{cgR\hat{\varepsilon}^2}{2(1 + \rho)}\]  

(37)

The expected profits of a type-\(\hat{\varepsilon}\) entrepreneur change from (15) to:

\[\Pi = \left(\frac{1 - \tau}{1 - \phi(\cdot)}(\hat{\varepsilon}R - \delta - \rho) - \frac{cg\hat{\varepsilon}^2}{2(1 - \phi(\cdot))} + 1 + \rho\right)E\]  

(38)

so:

\[\frac{d\Pi}{d\hat{\varepsilon}} = \left(\frac{\Delta(\hat{\varepsilon}, R, \tau, c)}{1 - \phi(\cdot)}(1 - \tau)(\hat{\varepsilon}R - \delta - \rho) - \frac{cR\hat{\varepsilon}^2}{2}\right)E\]  

(39)

Lemma 2 still applies, and therefore so do conditions (19) and (20) characterizing the entrepreneur’s choice of \(\hat{\varepsilon}\). The extensive-margin choices of entrepreneurs are still guided by (21). However, Lemma 5 for the cutoff value of \(R\) now becomes:

**Lemma 8** The cutoff value of \(R\) when monitoring costs are non-deductible satisfies

\[(1 - \tau)(\hat{\varepsilon}R - \delta - \rho) - \frac{cR\hat{\varepsilon}^2}{2} = 0\]  

(40)
To determine the intensive and extensive margin outcomes, consider first the marginal entrepreneur $\hat{R}$ for whom (40) applies. Substituting this into (39), we obtain:

$$\frac{d\Pi}{d\hat{\varepsilon}} = -\frac{c\hat{R}g\hat{\varepsilon}}{1-\phi}E < 0$$

This implies that $\hat{\varepsilon} = 0$ for $R = \hat{R}$, so $\hat{R}$ satisfies $\hat{\varepsilon} = \hat{\varepsilon}$ and is independent of $\tau$. Thus, the marginal entrepreneur takes no bankruptcy risk, which implies the extensive-margin decision is not affected by a cash-flow tax.

Eq. (39) characterizes the leverage decision for an inframarginal entrepreneur, where $d\Pi/d\hat{\varepsilon} = 0$ if $\hat{\varepsilon}^*$ is in the interior. Differentiating $\phi(\cdot)$ gives (29) for $d\phi/d\tau$, as in the base case. To evaluate the sign of $d\phi/d\tau$ in this case where bankruptcy costs are not deductible, it is useful to derive the following lemma.

**Lemma 9** Assume $\hat{\varepsilon}^*$ is in the interior. Then,

$$\frac{d\hat{\varepsilon}^*}{d\tau} < 0 \quad \text{as} \quad c > 0$$

When $\hat{\varepsilon}^*$ is in the interior, $\phi_{\hat{\varepsilon}} > 0$ by (17) and (20), while $\phi_{\tau} \geq 0$ by (37). Therefore, $d\phi/d\tau$ in (29) is smaller in value than in the base case, so this form of cash-flow tax is less effective as a corrective device. Moreover $d\phi/d\tau$ might even be negative in which case the outcome is further away from the social optimum than in the no-tax case. It is straightforward to show that the effect of $\tau$ on expected after-tax profits of the firm and on expected tax revenues are both ambiguous.

The results for this case can be summarized in the following proposition.

**Proposition 2** If banks cannot deduct monitoring costs,

i. Marginal entrepreneurs assume no bankruptcy risk;

ii. Entry is not affected by a cash-flow tax;

iii. Bankruptcy risk for infra-marginal entrepreneurs falls with the tax rate; and

iv. The change in leverage is smaller than in the base case, and may actually be negative so the tax may move the equilibrium away from the social optimum.
6.2 ACE Tax Base

Cash-flow taxation is often regarded as challenging for governments because it generates negative tax liabilities for investing firms and differs substantially from standard corporate income tax systems. In addition, in our context fully effective cash-flow taxation calls for deductions for ex post monitoring costs incurred by banks, which may be difficult to implement. ACE tax system achieves the same outcome in present value terms as the base-case cash-flow tax since monitoring costs are deductible from the banks’ tax base. As with cash-flow taxation, taxes under the ACE system are paid by the firm as long as it is profitable, but if it goes bankrupt the bank claims a tax credit for unclaimed capital deductions. In addition, since the ACE applies to both real and financial income, banks are liable to pay the tax. The form of the tax on profitable firms and banks and the tax credit the banks obtain for bankrupt firms are as follows.

For non-bankrupt firms, the end-of-period ACE tax liability is:

\[ \tilde{T}_{ACE} = \tau (\tilde{\epsilon} RK - K - \rho E - rB + (1 - \delta)K) \]

where the deduction for investment \( K \) occurs at the end of the period, while the firm gets to deduct costs of financing by equity and debt at the rates \( \rho \) and \( r \) respectively. The last term is the taxation of the sale of depreciated final assets. This expression simplifies to:

\[ \tilde{T}_{ACE} = \tau (\tilde{\epsilon} RK - \rho E - rB - \delta K) \text{ for } \epsilon \geq \tilde{\epsilon} \] (41)

In the case of bankruptcy, the firm does not repay its debt to the bank, and there is an unclaimed tax credit on its equity-financed investment of \( \tau (1 + \rho)E \). This amount is refunded to the bank. The bank also gets the returns on investment of the bankrupt firm.

The ex-post profit of the firm when \( \epsilon \geq \tilde{\epsilon} \) is (3) as above, which using (41) becomes:

\[ \tilde{\Pi} = (1 - \tau)(\tilde{\epsilon} RK + (1 - \delta)K - (1 + r)B) + \tau (1 + \rho)E \] (42)

Bankruptcy again occurs when \( \tilde{\Pi} < 0 \), where bankruptcy risk \( \tilde{\epsilon} \) at \( \tilde{\Pi} = 0 \) satisfies:

\[ 0 = \tilde{\epsilon} RK + (1 - \delta)K - (1 + r)B + \frac{\tau (1 + \rho)}{1 - \tau}E \] (43)

The banks pay the ACE tax on their financial income plus any revenues they obtain from bankrupt firms. The expected tax liability of a bank from a loan \( B \) under the ACE is:

\[ \tilde{T}_{ACE}^{B} = \tau \left( \int_{\tilde{\epsilon}}^{\tilde{\epsilon}_{max}} ((1 + r)B - (1 + \rho)B) gd\tilde{\epsilon} \right. \]

\[ + \left. \int_{0}^{\tilde{\epsilon}} ((\tilde{\epsilon} RK + (1 - \delta)K - (1 + \rho)E - (1 + \rho)B - c\tilde{\epsilon} RK) gd\tilde{\epsilon} \right) \] (44)
The first term of the tax base is net financial income when the loan is repaid. The second includes the net revenues from the bankrupt firms less the unclaimed tax credit owing to those firms, the deduction for the cost of repaying deposits and the cost of monitoring. Using (44), the bank’s zero-expected profit condition can be written:

\[(1 - \tau)(1 + \rho)B = (1 - \tau) \left( \int_{\hat{\varepsilon}^1}^{\hat{\varepsilon}} (1 + r)Bgd\hat{\varepsilon} + \int_{0}^{\hat{\varepsilon}} (\hat{\varepsilon}RK + (1 - \delta)K + \frac{\tau(1 + \rho)}{1 - \tau}E)gd\hat{\varepsilon} - cRKg\hat{\varepsilon}^2 \right) \]

Using (45) in (44), expected tax liabilities of the bank simplify to:

\[T_B^{ACE} = - \int_{0}^{\hat{\varepsilon}} \frac{\tau}{1 - \tau} (1 + \rho)Egd\hat{\varepsilon} \]

Though this is negative in expected terms, it will be positive if the firm does not go bankrupt. Using (43) to eliminate \((1 + r)B\) from (45) we obtain after routine simplification the same expression for leverage (12) as in the cash-flow tax base case and its properties in (13).

Using (42), expected profits of a type−R entrepreneur are:

\[\Pi = \int_{\hat{\varepsilon}}^{\varepsilon_{\text{max}}} \Pi g d\hat{\varepsilon} = \int_{\hat{\varepsilon}}^{\varepsilon_{\text{max}}} (1 - \tau)(\hat{\varepsilon}RK + (1 - \delta)K - (1 + r)B) + \tau(1 + \rho)E)gd\hat{\varepsilon} \]

Substituting (45) into (47) to eliminate \(\int_{\hat{\varepsilon}}^{\varepsilon_{\text{max}}} (1 + r)Bgd\hat{\varepsilon}\) and using \(B + E = K = E/(1 - \phi)\), we obtain:

\[\Pi = \frac{1 - \tau}{1 - \phi} \left( \varepsilon R - \delta - \rho - cgR\hat{\varepsilon}^2 \right) E + (1 + \rho)E \]

This is the same as (15) in the cash-flow tax base case. Lemma 6 still applies, so \(\hat{\varepsilon}\) is independent of \(\tau\). With \(\varepsilon\) independent of \(\tau\), Lemma 5 implies that \(\hat{\varepsilon}\) is also independent of \(\tau\). Then by (30), \(d\phi/d\tau = (1 - \phi)/(1 - \tau) = 1/D\), so \(D\) is independent of \(\tau\) as before. Also, \(\pi\) is independent of \(\tau\) by (32) since \(D\) is independent of \(\tau\) by footnote 6.

Finally, using (41) and (46), expected tax revenues from a type−R entrepreneur become:

\[T_R = \int_{\hat{\varepsilon}}^{\varepsilon_{\text{max}}} \tau(\hat{\varepsilon}RK - \rho E - rB - \delta K)gd\hat{\varepsilon} - \int_{0}^{\hat{\varepsilon}} \frac{\tau}{1 - \tau} (1 + \rho)Egd\hat{\varepsilon} \]

Using the bank zero-profit condition (45) to eliminate \(rB\), this expression reduces to

\[T_R = \int_{0}^{\varepsilon_{\text{max}}} \tau(\hat{\varepsilon}R - \rho - \delta)Kgd\hat{\varepsilon} - \tau cRg\hat{\varepsilon}^2 \]

which is equivalent to expected taxes for a type−R entrepreneur reflected in \(T\) for the cash-flow tax case (33). Thus, the ACE tax has the same effect on entrepreneurial behavior and on efficiency as the cash-flow tax with monitoring costs deductible. However, arguably the ACE is simpler to implement since monitoring costs are automatically deductible when banks are taxed.
The equivalence between the ACE and cash-flow taxation depends on the tax credit for bankrupt firms accruing to the banks. If we assume instead, following Bond and Devereux (2003), that the tax credit \( \tau(1 + \rho)E \) goes to shareholders in the event of bankruptcy, the equivalence breaks down. Ex post profits of the firm are still (42) if \( \varepsilon \geq \hat{\varepsilon} \), but in the event of bankruptcy profits are \(-\tau(1 + \rho)E\), so expected profits in (47) become

\[
\Pi = \int_{\hat{\varepsilon}}^{\varepsilon_{\text{max}}} \tilde{\Pi} g d\tilde{\varepsilon} = \int_{\hat{\varepsilon}}^{\varepsilon_{\text{max}}} (1 - \tau)(\tilde{\varepsilon}RK + (1 - \delta)K - (1 + \tau)B) g d\tilde{\varepsilon} + \tau(1 + \rho)E
\]

Condition (43) determining \( \hat{\varepsilon} \) still applies, but the bank’s zero expected profit condition (45) becomes

\[
(1 + \rho)B = \int_{\hat{\varepsilon}}^{\varepsilon_{\text{max}}} (1 + r)Bg d\tilde{\varepsilon} + \int_0^{\varepsilon_{\text{max}}} (\tilde{\varepsilon}RK + (1 - \delta)K) g d\tilde{\varepsilon} - cRKg \frac{\hat{\varepsilon}^2}{2}
\]

Using this zero-profit condition to eliminate \((1 + r)B\) from the expression for \(\Pi\) leads to (15) as above. From this we can infer that the extensive-margin decision is unaffected by the tax. However, if the bank’s zero-profit condition is combined with (43), the expression for \(\phi\) that results differs from (12), and as a result changes in \(\tau\) will affect \(\hat{\varepsilon}\) unlike in the cash-flow tax case. Thus, an ACE tax in which the tax credit on bankruptcy goes to the firm will not be equivalent to a cash-flow tax.\(^8\)

We summarize these results on the ACE as follows.

**Proposition 3** Under the ACE system, the tax is equivalent to the cash-flow tax with bankruptcy costs deductible as long as unclaimed tax credits on bankrupt firms accrue to the banks. Therefore, the ACE tax system increases leverage without affecting bankruptcy risk and expected profits and leads to higher expected tax revenues, rents and social surplus. This equivalence does not apply if the tax credits go to bankrupt firms instead of the banks.

The standard corporate tax is equivalent to the ACE defined in (41) without the cost of equity \(\rho E\) deducted. The analysis of the effect of the corporate tax on entry and investment turns out to be very complex so we do not report it here. However, some qualitative results are apparent. Since deductions for the cost of investment are less than under the ACE, some investments that would be undertaken under the latter are not profitable under the corporate tax. Therefore, the corporate tax is not as effective at correcting the market failure due to asymmetric information as the ACE is. Nonetheless, the corporate tax still acts as a device for the government to share the gains and losses of risky investments and therefore can still encourage leverage albeit less than with the ACE.

\(^8\)Bond and Devereux (2003) find that wind-up and bankruptcy decisions are not affected by the ACE when the tax credit goes to the firm. However, they do not have an intensive-margin decision in their model.
7 Risk-Averse Entrepreneurs

Assume now that entrepreneurs are risk-averse and unable to insure their uncertain project outcomes. Assume also that the government deploys a cash-flow tax with monitoring costs deductible as in our basic model. The key new element in this setting is that because the tax system cannot distinguish returns to risk from rents, the cash-flow tax necessarily applies to both.

Recall that all consumption takes place at the end of the period. Entrepreneurs who enter the risky industry invest all their wealth in their firm at the beginning of the period and consume the after-tax profits $\bar{\Pi}$ at the end, where the latter are given by (5) or (7). As in the base case with risk neutrality, entrepreneurs get no income in the event of bankruptcy since that goes to the bank. Let their end-of-period expected utility $V$ be:

$$V = \frac{1}{1 - \gamma} \int_{\hat{\epsilon}}^{\epsilon_{\text{max}}} \bar{\Pi}^{1-\gamma} g d\hat{\epsilon}$$

where $0 < \gamma < 1$, so the entrepreneur’s utility function exhibits constant relative risk aversion. Using (7) and (2), expected utility becomes:

$$V = \frac{(1 - \tau)RK}{1 - \gamma} \int_{\hat{\epsilon}}^{\epsilon_{\text{max}}} (\hat{\epsilon} - \hat{\epsilon})^{1-\gamma} g d\hat{\epsilon} = \frac{E^{1-\gamma}}{1 - \gamma} \left( R(1 - \tau) \right)^{1-\gamma} (\epsilon_{\text{max}} - \hat{\epsilon})^{2-\gamma} \frac{g}{2 - \gamma}$$

Alternatively, using (18) for $\bar{\pi}(\hat{\epsilon}, R, \tau, c)$, expected utility in (49) may be written:

$$V = \frac{E^{1-\gamma}}{1 - \gamma} \bar{\pi}(\hat{\epsilon}, R, \tau, c)^{1-\gamma} (\epsilon_{\text{max}} - \hat{\epsilon})^{2-\gamma} \frac{g}{2 - \gamma}$$

These alternative representations of expected utility will be useful in what follows.

As before, the entrepreneur decides whether to enter, and if so, how much to borrow and therefore how much risk to take on. Consider these decisions in reverse order.

7.1 Intensive-margin decision

The choice of leverage $\phi$ is again equivalent to the choice of bankruptcy risk $\hat{\epsilon}$ through (12). Differentiating (49) with respect to $\hat{\epsilon}$ and using the definition of $\Delta(\cdot)$ in (17), we obtain after straightforward simplification:

$$\frac{dV}{d\hat{\epsilon}} \propto \Delta(\hat{\epsilon}, R, \tau, c) - \frac{2 - \gamma}{1 - \gamma} \frac{1}{\epsilon_{\text{max}} - \hat{\epsilon}}$$

If the optimal choice of $\hat{\epsilon}$, $\hat{\epsilon}^*$, is in the interior, $dV/d\hat{\epsilon} = 0$, and (51) gives:

$$\Delta(\hat{\epsilon}^*, R, \tau, c) = \frac{2 - \gamma}{1 - \gamma} \frac{1}{\epsilon_{\text{max}} - \hat{\epsilon}^*} > 0$$

From (52) we obtain the following analogue to Lemma 6.
Lemma 10 If $\hat{\varepsilon}^*$ is in the interior, then
\[
\frac{d\hat{\varepsilon}^*}{d\tau} = 0 \quad \text{and} \quad \frac{d\hat{\varepsilon}^*}{d\gamma} < 0
\]

Note that Lemma 4 applies here as well, so bankruptcy risk and therefore leverage are increasing in entrepreneurial productivity: $d\hat{\varepsilon}^*/dR > 0$.

7.2 Extensive-margin decision

The entry decision involves comparing expected utility as an entrepreneur with that obtained from the safe alternative, where end-of-period consumption is $(1+\rho)E$. Entrepreneurs will enter if $V \geq ((1+\rho)E)^{1-\gamma}/(1-\gamma)$. From (49), $V$ is increasing in $R$, so the cutoff value $\hat{R}$ will be uniquely determined.

To characterize the extensive-margin decision, we show first that in equilibrium $\hat{\varepsilon}^* = 0$ for the marginal entrepreneur, $\hat{R}$. Let $R_0$ be the value of $R$ such that $\hat{\varepsilon}^*$ just becomes zero. Then, $\hat{R}$ satisfies the following:

Lemma 11 $\hat{R} \leq R_0$ as $\gamma \geq 0$

The implication of Lemma 11 is that $\hat{\varepsilon}^* = 0$ at $R = \hat{R}$ when $\gamma \geq 0$, and this applies regardless of the values of $c$ or $\tau$. Note that this confirms the result that we found earlier in the risk-neutral model with $\gamma = 0$ where the marginal entrepreneur choose $\hat{\varepsilon}^* = 0$ as well.

In the risk-neutral case, $\hat{R} = R_0$, so $\hat{\varepsilon}^* > 0$ for all inframarginal entrepreneurs.

Lemma 11 implies that with $\gamma > 0$, $\hat{\varepsilon}^* = 0$ for all entrepreneurs with $R \in [\hat{R}, R_0]$. In that range, there is no risk of bankruptcy, so $r = \rho$ by the banks’ zero-profit conditions (11) and ex post after-tax profits in (5) can be written, using $B = K - E = E/(1-\phi) - E$:

\[
\tilde{\Pi} = \left(\frac{1-\tau}{1-\phi} (\hat{\varepsilon}R - \delta - \rho) + 1 + \rho\right)E = (D(\hat{\varepsilon}R - \delta - \rho) + 1 + \rho)E
\]

where, recall, $D \equiv (1-\tau)/(1-\phi)$. Therefore, expected utility can be written:

\[
V = \int_{\varepsilon_{\max}}^{1-\gamma} \frac{\tilde{\Pi}^{1-\gamma}}{1-\gamma} g d\tilde{\varepsilon} = \frac{E^{1-\gamma}}{1-\gamma} \int_{\varepsilon_{\max}}^{1-\gamma} (D(\hat{\varepsilon}R - \delta - \rho) + 1 + \rho)^{1-\gamma} g d\tilde{\varepsilon} \quad (53)
\]

The value of $\hat{R}$ is determined where $V = (1+\rho)^{1-\gamma}E^{1-\gamma}/(1-\gamma)$ or using (53),

\[
\int_{\varepsilon_{\max}}^{1-\gamma} (D(\hat{\varepsilon}\hat{R} - \delta - \rho) + 1 + \rho)^{1-\gamma} g d\tilde{\varepsilon} = (1 + \rho)^{1-\gamma} \quad (54)
\]

Entrepreneurs will enter the risky sector only if $R \geq \hat{R}$. Those with $R < \hat{R}$ will invest their wealth in a safe asset.
The expression for $V$ in (53), which applies for all $R \in [\hat{R}, R_0]$, has a further implication for the intensive margin. In this range, an entrepreneur of given $R$ will choose leverage, or equivalently $D \equiv (1 - \tau)/(1 - \phi)$, to maximize $V$. The first-order condition $\partial V/\partial D = 0$ reduces to:

$$
\int_{0}^{\varepsilon_{\text{max}}} (D^*(\hat{\varepsilon} R - \delta - \rho) + 1 + \rho)^{-\gamma} g d\hat{\varepsilon} = 0
$$

(55)

where $D^*$ is the optimal choice of $D$. This value of $D^*$, and therefore $\phi$ will be increasing in $R$. As $R \to R_0$, $\phi \to (1 - \tau)(1 - \delta)/(1 + \rho) + \tau$, which is the value of $\phi$ that just satisfies (12) when $\hat{\varepsilon} = 0$.

### 7.3 The effects of cash-flow taxation

We next turn to the effects of the base-case cash-flow tax on the entrepreneurs’ intensive- and extensive-margin decisions when entrepreneurs are risk-averse. It is straightforward to show that the same effects occur under the ACE tax when tax credits on unused capital costs go to the banks.

#### 7.3.1 Intensive margin decision

By Lemma 11, since $\gamma > 0 \hat{R} < R_0$. Consider first entrepreneurs in the range $R \in [\hat{R}, R_0]$. As we have seen, for these entrepreneurs, $\hat{\varepsilon}^* = 0$ and the first-order condition (55) again applies. This yields a unique value of $D^*$ for each $R$ which is independent of the tax rate. Since $D^* = (1 - \tau)/(1 - \phi)$, leverage will rise by the same amount as an increase in $\tau$. And, since $D^*$ is invariant with the tax rate, so will $V$ be as seen by (53). Thus, for entrepreneurs in this range, a result analogous to Domar-Musgrave (1944) applies. Entrepreneurs offset an increase in $\tau$ by increasing private risk-taking, while achieving the same level of expected utility $V$. The increase in leverage $\phi$ entails more investment and therefore more rent generation, which accrues to the government in increased tax revenues as discussed below.

For entrepreneurs of type $R > R_0$, $\hat{\varepsilon}^* > 0$. Their intensive-margin decision is similar to the base case with risk-neutrality above. Eq. (29) again applies, and since $d\hat{\varepsilon}^*/d\tau = 0$ by Lemma 10 (29) simplifies to $d\phi/d\tau = \phi_\tau$. By (13), $\phi_\tau = (1 - \tau)/(1 - \phi)$, so $\tau$ encourages leverage. However, the increase in leverage does not translate into an increase in bankruptcy risk $\hat{\varepsilon}$, since $d\hat{\varepsilon}^*/d\tau = 0$. As well, as we have shown in footnote 6, $D = (1 - \tau)/(1 - \phi)$ is constant when $\phi_\tau = (1 - \tau)/(1 - \phi)$. Therefore, with $D$ and $\hat{\varepsilon}$ constant, so is expected utility $V$ in (49) as long as leverage $\phi$ and $K$ are in the interior. This again is analogous to the Domar and Musgrave (1944) result: a tax on capital income with full loss-offset encourages risk-taking by risk-averse individuals because the government is sharing the risk with the entrepreneur on actuarially fair terms. In the case of a cash-flow tax, the government is
sharing the risk of the entrepreneur, and as a consequence private risk \( \hat{\varepsilon} \) does not change as a result of the tax.

### 7.3.2 Extensive margin decision

Since \( \hat{R} < R_0 \) we have that \( \hat{\varepsilon}^* = 0 \) for the marginal entrepreneur. For a given tax rate \( \tau \), the marginal entrepreneur \( \hat{R} \) chooses leverage such that the first-order condition (55) is satisfied at \( R = \hat{R} \). This leads to a unique value of the optimal \( D^* = (1 - \tau)/(1 - \phi) \) regardless of the tax rate. Recall that \( \hat{R} \) satisfies (54). Since the choice of \( D^* \) by the marginal entrepreneur is independent of the tax rate, the value of \( \hat{R} \), and therefore the extensive margin decision, that satisfies (54) is independent of \( \tau \).

### 7.3.3 Expected profits and tax revenues

As well as \( V \) being unaffected by the cash-flow tax, so are expected profits \( \Pi \). To see this, consider the expression for the expected rate of return on equity, \( \bar{\pi} \), in (18). Since both \( \hat{\varepsilon}^* \) and \( D^* = (1 - \tau)/(1 - \phi) \) are independent of \( \tau \), \( \bar{\pi} \) will also be invariant with \( \tau \). Therefore, entrepreneurs are able to offset the effect of the tax on their expected after-tax profits by increasing their leverage. The increase in leverage will correspond to an increase in borrowing and investment, which in turn will increase expected before-tax profits and therefore expected tax revenue to the government since after-tax profits are unchanged.

To see that expected tax revenues will rise, note that (33) still applies, and the change in expected tax revenues will again be given by (34), which is positive since \( d\phi/d\tau > 0 \). Expected tax revenues rise due both to a mechanical effect and to an increase in the tax base because of increased leverage and investment. Social welfare will increase if the increase in expected tax revenues is valuable to the government, and that depends on how the government evaluates the increase in risk that might accompany the tax revenues.

Our findings with risk-averse entrepreneurs are summarized as follows.

**Proposition 4** With risk-averse entrepreneurs subject to cash-flow taxation with monitoring costs deductible, equilibrium has the following properties:

i. There is a range of entrepreneurs with \( R \in [\hat{R}, R_0] \) for whom there is no risk of bankruptcy, so \( \hat{\varepsilon}^* = 0 \) and \( r = \rho \).

ii. For all entrepreneurs with \( \hat{\varepsilon}^* \) below the maximum, leverage increases with the cash-flow tax rate, while bankruptcy risk, expected profits and expected utility are unchanged.

iii. The cash-flow tax is neutral with respect to entry decisions.
iv. Expected tax revenues increase with the tax rate

8 Concluding Remarks

In this paper, we analyzed the impact of cash-flow business taxation on firms’ choice of leverage and on decisions to enter a risky industry in a setting where entrepreneurs may be risk-averse and face bankruptcy risk, and where there is asymmetric information between entrepreneurs and financial intermediaries. The neutrality of cash-flow taxation found by Bond and Devereux (1995, 2003) in the absence of asymmetric information and risk-aversion no longer applies under these features. Moreover, there is too little investment in the information-constrained social optimum compared with the full-information case. The issue is then whether cash-flow taxation improves social efficiency.

With risk-neutral entrepreneurs, cash-flow taxation taxes rents only. Without asymmetric information in the credit market (i.e. with costless monitoring), the cash-flow tax affects neither entry nor leverage decisions, so the standard neutrality results apply. CAA and RRT systems are also neutral in this case provided that the risk-free interest rate is used to carry forward untaxed cash flows. When banks must undertake costly monitoring of firms that declare bankruptcy, the tax does not affect entry decisions, given that the marginal entrepreneur earns no rent, but it affects leverage. If banks can obtain a tax deduction for the monitoring costs they incur on the bankrupt firms they seize, the tax will increase leverage but will leave bankruptcy risk and the firms’ after-tax profits unchanged. The tax will actually increase social welfare in this case. By inducing firms to increase leverage, and therefore capital, the tax leads to higher pre-tax profits, or rents, without affecting bankruptcy risk. These additional pre-tax profits are taxed away by the government leading to a higher constrained social surplus. By inducing more investment, the cash-flow tax is implicitly correcting for the inefficiencies of the information-constrained outcome.

An ACE tax in which firms can deduct actual interest payments can be designed that is equivalent to the cash-flow tax with monitoring costs deductible. With such a system no tax is applied to bankrupt firms, but the banks obtain a tax credit for unused deductions by the firm. The ACE tax has the advantage over cash-flow taxation that monitoring costs incurred by the banks are automatically deducted from the tax in the event of bankruptcy since banks are subject to the ACE.

With risk-averse entrepreneurs, cash-flow taxation taxes both rents and return to risk. Without monitoring costs, or if monitoring costs are deductible, the cash-flow tax increases leverage, but is neutral with respect to bankruptcy risk, expected profits and expected utility. The cash-flow tax involves some risk-sharing between the government and firms,
and increases social welfare in this case for firms will levels of capital in the interior. The tax increases rents and tax revenues without affecting expected utility.

We have assumed that asymmetric information involved moral hazard rather than adverse selection, so banks can observe firm types but not their profits. If banks cannot observe the productivity of entrepreneurs ex ante, there will be an adverse selection problem as in Stiglitz and Weiss (1981) and de Meza and Webb (1987), among others. In this case, if banks cannot offer separating contracts, all entrepreneurs face the same interest rate. In contrast to the case considered here, the equilibrium without taxation will be inefficient along the extensive margin and cash-flow taxation will generally not be neutral. In particular, there will be excessive entry by the least-productive entrepreneurs to take advantage of the favorable interest rate. A cash-flow tax discourages entry, thereby improving efficiency. It would be interesting to extend the analysis to the case where banks are able to offer contracts in which the interest rate varies with the size of loan, so firms can be separated by type. There will be informational rents that might influence the effect of cash-flow taxes.

Appendix
Proof of Lemma 1
Using (6), the righthand side of (11) can be written:

\[
((1 - \tau)(\hat{\varepsilon}R + (1 - \delta)) + \tau(1 + \rho))K(1 - g\hat{\varepsilon}) + (1 - \tau)RK\frac{g\hat{\varepsilon}^2}{2}
\]

\[
+ ((1 - \tau)(1 - \delta) + \tau(1 + \rho))Kg\hat{\varepsilon} - (1 - \tau)\frac{cRKg\hat{\varepsilon}^2}{2}
\]

\[
= (1 - \tau)\left(\hat{\varepsilon}(1 - g\hat{\varepsilon}) + \frac{g\hat{\varepsilon}^2}{2}\right)RK + ((1 - \tau)(1 - \delta) + \tau(1 + \rho))K - (1 - \tau)\frac{cRKg\hat{\varepsilon}^2}{2}
\]

\[
= (1 - \tau)\left(1 - \frac{g\hat{\varepsilon}}{2} - \frac{cR\hat{\varepsilon}}{2}\right)\hat{\varepsilon}RK + ((1 - \tau)(1 - \delta) + \tau(1 + \rho))K
\]

Proof of Lemma 2
Rewrite \(\pi\) in (15) as

\[
\pi(\hat{\varepsilon}, R, \tau, c) = \frac{1}{1 - \phi} \left( (1 - \tau)(R\overline{\varepsilon} - \delta - \rho - \frac{cR\overline{\varepsilon}^2}{2}) + (1 + \rho)(1 - \phi) \right)
\]

From (12), we obtain

\[
(1 + \rho)(1 - \phi) = -(1 - \tau)\left(1 - \frac{g\hat{\varepsilon}}{2} - \frac{cR\hat{\varepsilon}}{2}\right)\hat{\varepsilon}R + (1 - \tau)(\rho + \delta)
\]

Substituting this in the expression for \(\pi\) gives, using \(\overline{\varepsilon} = \varepsilon_{\text{max}}/2\) and \(\varepsilon_{\text{max}} = 1/g\):

\[
\pi(\hat{\varepsilon}, R, \tau, c) = \frac{1 - \tau}{1 - \phi} R\left(\overline{\varepsilon} - \hat{\varepsilon}\left(1 - \frac{g\hat{\varepsilon}}{2}\right)\right) = \frac{1 - \tau}{1 - \phi} \frac{R\overline{\varepsilon}}{2} (\hat{\varepsilon} - \varepsilon_{\text{max}})^2
\]
Proof of Lemma 4
Differentiate $\frac{d \Pi}{d \hat{\varepsilon}} = 0$ in (19), and use $\Delta_R = \phi_{\varepsilon R}/(1 - \phi) + \phi_{\varepsilon} \phi_R/(1 - \phi)^2 > 0$ and the second order-conditions on $\hat{\varepsilon}$. \hfill \blacksquare

Proof of Lemma 6
Differentiating $\Delta(\hat{\varepsilon}, \tau, c)$, we have $\Delta_{\tau} = \phi_{\varepsilon \tau}/(1 - \phi) + \phi_{\tau} \phi_{\varepsilon}/(1 - \phi)^2$. Using (13) for $\phi_{\tau}$, $\phi_{\varepsilon}$ and $\phi_{\varepsilon \tau}$,

$$\Delta_{\tau} = - \frac{1 - g \hat{\varepsilon} - cg \hat{\varepsilon}}{(1 + \rho)(1 - \phi)} R + \frac{1 - \phi}{1 - \tau} \frac{1}{1 + \rho} (1 - g \hat{\varepsilon} - cg \hat{\varepsilon}) R \frac{1}{(1 - \phi)^2} = 0 \quad (56)$$

Differentiate the first-order condition (20) to obtain:

$$\Delta_{\tau} d\tau + \left( \Delta_{\hat{\varepsilon}} - \frac{2}{(\hat{\varepsilon}_{\max} - \hat{\varepsilon}^*)^2} \right) d\hat{\varepsilon} = 0$$

The coefficient of $d\hat{\varepsilon}$ is negative by the second-order condition, and $\Delta_{\tau} = 0$ by (56). \hfill \blacksquare

Proof of Lemma 7
Differentiate (8) with respect to $\tau$ and use $\frac{d \hat{\varepsilon}}{d \tau} = 0$ by Lemma 6 to obtain:

$$- \left( \hat{\varepsilon} R + (1 - \delta) - (1 + \rho) + (1 + r) \phi_{\tau} \right) d\tau - \phi dr = 0$$

Since $\phi_{\tau} = (1 - \phi)/(1 - \tau)$ by (13), this becomes:

$$\frac{dr}{d\tau} = - \frac{1}{\phi} \left( \hat{\varepsilon} R - (\delta + \rho) + (1 + r) \frac{1}{1 - \tau} \right)$$

Using (8), this can be written:

$$\frac{dr}{d\tau} = - \frac{r - \rho}{\phi 1 - \tau}$$

which is negative since $r > \rho$ by the bank’s zero profit condition. \hfill \blacksquare

Proof of Lemma 9
Differentiate the first-order condition (20) to obtain:

$$\Delta_{\tau} d\tau + \left( \Delta_{\hat{\varepsilon}} - \frac{2}{(\hat{\varepsilon}_{\max} - \hat{\varepsilon}^*)^2} \right) d\hat{\varepsilon} = 0$$

The coefficient of $d\hat{\varepsilon}$ is negative by the second-order condition. Since $\Delta > 0$ by (20), $\phi_{\varepsilon} > 0$ by the definition of $\Delta$ in (17). Differentiating (17) by $\tau$, we have $\Delta_{\tau} = \phi_{\varepsilon \tau}/(1 - \phi) + \phi_{\tau} \phi_{\varepsilon}/(1 - \phi)^2$. Using (37) for $\phi_{\tau}$ and $\phi_{\varepsilon \tau}$,

$$\Delta_{\tau} = - \frac{1}{1 - \phi} \frac{R}{1 + \rho} (1 - g \hat{\varepsilon}) + \frac{\phi_{\varepsilon}}{1 - \phi} \frac{1}{1 - \tau} - \frac{\phi_{\varepsilon}}{(1 - \phi)^2} \frac{1}{1 - \tau} \frac{c R g \hat{\varepsilon}^2}{2(1 + \rho)}$$
Using (37) for $\phi_\varepsilon$, we obtain:

$$\Delta_r = -\frac{c\varepsilon R g}{(1 + \rho)(1 - \tau)(1 - \phi(\cdot))} \left(1 + \frac{\phi_\varepsilon(\cdot) \varepsilon}{1 - \phi(\cdot)}\right)$$

(57)

Lemma 9 follows since $\Delta_r < 0$ as $c > 0$ by (57) when $\phi_\varepsilon > 0$.

**Proof of Lemma 10**

Differentiate the first-order condition (52) to obtain:

$$\Delta_r d\tau - \left(\frac{1}{\varepsilon_{\text{max}} - \hat{\varepsilon}^*(1 - \gamma)^2}\right)d\gamma + \left(\Delta_\varepsilon - \frac{2 - \gamma}{1 - \gamma (\varepsilon_{\text{max}} - \hat{\varepsilon}^*)^2}\right)d\hat{\varepsilon} = 0$$

The coefficient of $d\hat{\varepsilon}$ is negative by the second-order condition. Lemma 10 follows since $\Delta_r = 0$ by (56).

**Proof of Lemma 11**

Note first that the following lemma indicates when $\hat{\varepsilon}^* = 0$.

**Lemma 12**

$$\hat{\varepsilon}^* = 0 \quad \text{if} \quad \frac{\varepsilon R}{\delta + \rho} < \frac{1}{2}(1 - \gamma)$$

(58)

Proof: $\hat{\varepsilon}^* = 0$ if $dV/d\hat{\varepsilon}|_{\hat{\varepsilon}=0} < 0$, or $\Delta(0, R, \tau, c) < (2 - \gamma)/(1 - \gamma)2\pi$ by (51). By (17),

$$\Delta(0, R, \tau, c) = \frac{\phi_\varepsilon(0, R, \tau, c)}{1 - \phi(0, R, \tau, c)} = \frac{(1 - \tau)R/(1 + \rho)}{1 - (1 - \tau)(1 - \delta)/(1 + \rho) - \tau} = \frac{R}{\rho + \delta}$$

using (12) and (13).

Next, we can derive the conditions under which $\hat{\varepsilon}^* = 0$ for the marginal entrepreneur $\hat{R}$. From Lemma 12, $R_0$ satisfies:

$$\frac{\varepsilon R_0}{\delta + \rho} = \frac{1}{2}(1 - \gamma)$$

(59)

The expected utility of this entrepreneur, from (50) and using $\varepsilon_{\text{max}} = 1/g$, becomes:

$$V_0 = \frac{E^{1-\gamma} \pi(0, R_0, \tau, c)^{1-\gamma}}{1 - \gamma} \frac{2^{1-\gamma}}{2 - \gamma}$$

Using (18) for $\pi$, (12) for $\phi$, and (59), this may be written after some manipulation:

$$V_0 = \frac{(1 + \rho)^{1-\gamma} E^{1-\gamma}}{1 - \gamma} \frac{1}{(1 - \gamma)^{1-\gamma}(2 - \gamma)}$$

(60)
Recall that for the marginal entrepreneur, $\hat{R}$,

$$V(\hat{R}) = \frac{(1 + \rho)^{1-\gamma}E^{1-\gamma}}{1 - \gamma}$$

We have that $\hat{R} \leq R_0$ if $V(\hat{R}) \leq V_0$, or from (60) if $(1 - \gamma)^{1-\gamma}(2 - \gamma)^\gamma \leq 1$, or equivalently $\gamma \geq 0$, where the equality applies for $\gamma = 0$. \[\blacksquare\]
References


Panteghini, Paolo, Maria Laura Parisi, and Francesca Pighetti (2012), ‘Italys ACE Tax and
President’s Advisory Panel on Federal Tax Reform (2005), Simple, Fair and Pro-Growth: Proposals to Fix Americas Tax System (President’s Advisory Panel: Washington).