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Multi-Firm Mergers with Leaders and Followers[*]

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Abstract

This paper analyzes mergers involving several leaders and followers in Stackelberg models, with the merged entity acting as a leader. Adding a follower to a merger increases its profitability or reduces its losses. A merger between one leader and any number of followers is always profitable. When a merger involves two leaders, it requires a sufficiently large proportion of followers to participate in it to be profitable. A merger is less likely to be profitable when the number of participating leaders is intermediate and the number of participating followers is small. All mergers involving leaders and followers are welfare reducing. Overall, Stackelberg leadership partially alleviates the merger paradox.

Key words: Mergers, Merger profitability, Merger paradox, Stackelberg, Leaders, Followers.

JEL Classification: D43, L13.

Résumé

Ce papier analyse les fusions impliquant plusieurs meneurs et suiveurs dans les modèles de Stackelberg, où la firme fusionnée agit comme un meneur. Ajouter un suiveur à une fusion augmente sa profitabilité ou réduit ses pertes. Toute fusion entre un meneur et n'importe quel nombre de suiveurs est profitable. Lorsque deux meneurs participent à une fusion, il faut qu'un nombre suffisant de suiveurs s'y joignent pour qu'elle soit profitable. Une fusion a moins de chances d'être profitable lorsque le nombre de meneurs qui y participent est intermédiaire et le nombre de suiveurs est petit. Toutes les fusions impliquant des meneurs et des suiveurs réduisent le bien-être. En général, le leadership de Stackelberg allège en partie le paradoxe des fusions.

Mots clés : Fusions, Profitabilité des fusions, Paradoxe des fusions, Stackelberg, Meneurs, Suiveurs.

Classification JEL : D43, L13.

1. Introduction

The study of mergers has revealed, starting with Salant et al. (1983) but also going back to Stigler (1950), that mergers in a Cournot model may be unprofitable unless they involve a substantial proportion (80%) of firms in the industry. Because the merger entails an output expansion by outsiders, if the number of the latter is large relative to the number of insiders, this expansion can be large enough to make the merger unprofitable to the merging firms.

Yet, empirical evidence does not point to a systematic unprofitability of mergers. Scherer (1980) finds that on average the private gains from mergers are either negative or almost nil. Bruner (2002) reviews 130 empirical studies on merger profitability between 1971 and 2001, and concludes that the shareholders of target firms gain, while the returns to the shareholders of the buying firms are close to zero. From reviewing a large number of empirical studies, Budzinski and Kretschmer (2007) conclude that unprofitable mergers represent between 25% and 50% of mergers. In fact, bilateral mergers are observed in most industries (Office of Fair Trading, 1999); nothing suggests that the bulk of these mergers satisfy the high threshold (required for profitability) imposed by the static Cournot model. The bulk of the empirical evidence on mergers points in the same direction: mergers are profitable more often than what is suggested by the theoretical models.

Different mechanisms[1] have been proposed through which firms can evade the merger paradox,[2] namely, that "it is ... quite difficult to construct a simple economic model in which there are sizable gains for firms participating in a horizontal merger *that is not a merger to monopoly*" (Pepall et al., 2014:388). Of particular relevance to the present paper is the interaction between market leadership and the profitability of mergers. Almost all the papers mentioned in footnote 1 analyze mergers in a Cournot setting. The analysis can be different in a Stackelberg model, where some firms act as leaders and others as followers. For example, Mizutani (2010) shows that following the merger between *Japan Airlines* and *Japan Air System* in 2003, they became an equal rival to *All Nippon Airways*, and the competition changed from Stackelberg to Cournot. Romeo (2012) uses Stackelberg merger simulation in the beer industry, and finds that the post-merger price and market shares are very different from those obtained under Bertrand simulations.

A number of papers have looked at the interaction between Stackelberg leadership and merger profitability. Levy and Reitzes (1989) model merger between two firms (who become a leader) in a spatial setting. Levin (1990) analyzes mergers starting from a Cournot setting, but allows the merged firms to have a more general conjectural variation (for instance, become a Stackelberg leader); he finds that such a merger, if it induces a reduction in output of the merged entity, is profitable iff it involves firms with at least a 50% pre-merger market share. Higgins

[1] Access to scarce capital (Perry and Porter, 1985); product differentiation with Bertrand competition (Deneckere and Davidson, 1985); non-Cournot behavior (Kwoka, 1989); capacity constraints (Baik, 1995); properties of the demand function (Cheung, 1992; Faulí-Oller, 1997; Hennessy, 2000); choice of product range (Lommerud and Sφrgard, 1997); the short run vs. the long run (Polasky and Mason, 1998); spatial price discrimination (Rothschild et al., 2000); dynamic Cournot competition (Dockner and Gaunersdorfer, 2001); open-loop vs. closed-loop strategies (Benchekroun, 2003); improved information flows inside the merged entity (Huck et al., 2004); setting competition between the internal divisions of the merged firm (Creane and Davidson, 2004); intra-firm coordination (Higl and Welzel, 2005); cost-reducing alliance (Sawler, 2005); commitment to maintain pre-merger profits (Huck et al., 2007); relative performance rewards (Fan and Wolfstetter, 2014); product differentiation with cost asymmetries (Gelves, 2014); workers' cooperatives (Delbono and Lambertini, 2014). See Huck et al. (2005) for a survey.

[2] The term was first coined by Pepall et al. in the first edition of their textbook, but was first formalized by Salant et al. (1983).

(1996) analyzes mergers in a price-setting Stackelberg model, and finds that when the merger includes the leading firm, price will increase proportionally to the increase in the capacity of the leader. Huck et al. (2001), Feltovich (2001) and Kabiraj and Mukherjee (2003) find that two leaders (followers) have an incentive to merge iff there are no other leaders (followers); in addition, a merger between one leader and one follower is always profitable. Feltovich (2001) and Daughety (1990) consider the possibility that two followers merge to form a leader, and find that such a merger is always profitable. Gelves (2010) and Takarada and Hamada (2006) analyze mergers involving leaders and followers, but focus on mergers involving only two firms. Hamada and Takarada study mergers in a Stackelberg model, however they consider only cases where leaders merge with other leaders, or followers merge with other followers. Heywood and McGinty (2007) and Brito and Catalão-Lopes (2011) study the effect of cost convexity on the profitability of a merger of two firms. Escrihuela-Villar (2013) studies merger profitability between a leader and a follower, and shows that it depends on the extent of collusion among leaders. Cunha and Vasconcelos (2014) study the profitability of mergers between two firms in a Stackelberg model where the merger gives rise to efficiency gains. Le Pape and Zhao (2014) study mergers between two firms in a Stackelberg model in the presence of uncertainty about the post-merger costs.

One common assumption of the studies mentioned above is that only bilateral mergers are considered, whether the merger involves two leaders, two followers, or a follower and a leader. Yet, just as in a Cournot model, a merger may involve more than two firms. A complete analysis of the profitability of mergers in Stackelberg models requires the consideration of mergers between arbitrary numbers of firms, involving any number of leaders and followers. A few papers have started to consider mergers of more than two firms in Stackelberg models. Heywood and McGinty (2008) study merger between a leader and a number of followers in a Stackelberg model in the presence of cost convexity, and find that such mergers are always profitable. Escrihuela-Villar and Faulí-Oller (2008) model, among other types of mergers, mergers involving one leader and several followers, and study the role of cost asymmetry between leaders and followers. However, no study considers the general case analyzed here, where a merger my involve several leaders and several followers.

Extending this literature, the current paper sets up a Stackelberg model with leaders and followers. Three types of mergers are considered: mergers involving leaders only, followers only, and leaders and followers. For the latter type of merger, it is assumed the merged entity forms a leader firm. For mergers between firms of the same type, it is shown that the same critical threshold found in Cournot models is applicable; hence, a merger between leaders (followers) is profitable iff it involves 80% of leaders (followers).

The analysis is different, however, when a merger involves both leaders and followers. Adding a follower to a merger always increases its profitability or reduces its losses. A merger between one leader and any number of followers is always profitable. When a merger involves two leaders, it requires a sufficiently large proportion of followers to participate in it, and this proportion decreases with the number of followers and increases with the number of leaders present in the market. In particular, a merger can be profitable even when the number of leaders participating is smaller than the standard threshold, provided the number of followers participating in the merger is large enough (and provided there are enough followers around to make this possible). A merger is less likely to be profitable when the number of participating leaders is intermediate, and the number of participating followers is small. This is because with this configuration, the merged entity does not have enough market power (there are still many leaders around) to compensate for the lost profits of the participating leaders, and the small number of followers makes their contribution to the profitability of the merger insufficient to compensate for that disadvantage. When all followers participate to a merger, this merger is more likely to be profitable when the number of followers is large.

While mergers with followers are often appealing to leaders, the effects on consumers and welfare are less encouraging. Any merger involving at least one leader and one follower, with the merged entity acting as a leader, reduces consumer surplus and welfare, and increases total profits and the profits of each outsider. Overall, the paper shows that Stackelberg leadership partially alleviates the merger paradox.

Hence, this paper contributes to the literature on merger profitability by analyzing the profitability of mergers involving leaders and followers. It contributes to the literature on mergers between asymmetric firms by explicitly considering the asymmetric leader/follower roles in Stackelberg models. It extends the analysis of papers having considered mergers in Stackelberg models by considering not only bilateral mergers, but also mergers involving any number of leaders and followers.

The paper is organized as follows. The next section presents the basic model and analyzes the profitability of mergers involving leaders only or followers only. Section 3, which constitutes the core of the paper, analyzes mergers involving both leaders and followers. It is divided into three parts. The first part derives some general results regarding mergers involving leaders and followers; the second part looks at some specific market structures which illustrate the general results derived in the first part; the third part analyzes the welfare implications of such mergers. The last section concludes.

2. The basic model and mergers of leaders only or followers only

There are L leaders and F followers competing in a Stackelberg market. Market demand is given by $P(Y)=1-Y$, where $Y=Ly_L+Fy_F$ denotes total output, and y denotes the output of an individual firm. For simplicity, there are no production costs. The leaders determine their outputs simultaneously and, after observing the leaders' outputs, the followers choose their outputs. The profit of a leader is given by $\pi_L=P(Y)y_L$, while the profit of a follower is $\pi_F=P(Y)y_F$.

We start with the followers' profit maximization. Follower i solves the following problem:

$$\max_{y_{F,i}} \ \pi_{F,i} = \big(1 - y_{F,i} - (F-1)y_{F,-i} - Ly_L\big)y_{F,i} \ , \quad i = 1, \dots, F, \tag{1}$$

where $-i$ indicates all other firms of the same type. Taking the f.o.c., imposing symmetry and solving for y_F gives each follower's reaction function:

$$y_F(y_L) = \frac{1 - Ly_L}{F+1}. \tag{2}$$

Each leader anticipates this reaction function and solves the following problem:

$$\max_{y_{L,i}} \ \pi_{L,i} = \left(1 - y_{L,i} - (L-1)y_{L,-i} - F\left[\frac{1-y_{L,i}-(L-1)y_{L,-i}}{F+1}\right]\right)y_{L,i} \ , \qquad i = 1, \dots, L. \tag{3}$$

Solving this problem, imposing symmetry and substituting the solution into (2) yields outputs in the pre-merger symmetric equilibrium:

$$y_L^*(L) = \frac{1}{L+1}, \qquad y_F^*(L,F) = \frac{1}{(L+1)(F+1)}. \tag{4}$$

Substituting (4) into profits results in per firm profits as a function of the number of leaders and followers:

$$\pi_L^*(L,F) = \frac{1}{(L+1)^2(F+1)}, \qquad \pi_F^*(L,F) = \frac{1}{(L+1)^2(F+1)^2}. \tag{5}$$

Consider now how a merger can upset this equilibrium. We start with a merger involving only leaders. Let $N_L \in [2,L]$ leaders merge and form one firm; the number of leaders becomes L-N_L+1, while the number of followers is unchanged at F. Substituting these values into (5) yields the per firm post-merger profits:

$$\pi_L(L-N_L+1,F) = \frac{1}{(L-N_L+2)^2(F+1)}, \qquad \pi_F(L-N_L+1,F) = \frac{1}{(L-N_L+2)^2(F+1)^2}. \tag{6}$$

This merger is profitable iff the profits made by the merged firms after the merger exceed the profits they made before the merger:

$$\pi_L(L-N_L+1,F) > N_L \pi_L(L,F). \tag{7}$$

Substituting (5) and (6) into (7) and solving for N_L shows (7) is satisfied iff

$$N_L > N_L^c = \frac{2L+3-\sqrt{4L+5}}{2}, \tag{8}$$

where N_L^c denotes the critical threshold for profitability. Note that N_L^c does not depend on F. Moreover, this is the same critical threshold for the profitability of a merger in a Cournot model. Therefore, the presence of followers by itself does not alleviate the merger paradox: a substantial proportion (at least 80%) of leaders still need to merge to ensure profitability. Because the merger entails a contraction of the output of the outsider leaders, the number of merging leaders must be substantial enough for this expansion not to hurt the merged entity too much. Moreover, even though followers also expand their output following the merger by leaders (this is clear from equation 4), this does not change N_L^c relative to a Cournot model.

We next analyze the profitability of a merger involving only followers. Let $N_F \in [2,F]$ be the number of merging followers. There are now L leaders and F-N_F+1 followers after the merger. Note that contrary to Daughety (1990), we do not assume the merged entity becomes a leader. From (5) the profits following such a merger are given by

$$\pi_L(L,F-N_F+1) = \frac{1}{(L+1)^2(F-N_F+2)}, \qquad \pi_F(L,F-N_F+1) = \frac{1}{(L+1)^2(F-N_F+2)^2}. \tag{9}$$

This merger is profitable iff

$$\pi_L(L,F-N_F+1) > N_F \pi_F(L,F). \tag{10}$$

Using (5) and (6), this condition is satisfied iff

$$N_F > N_F^c = \frac{2F+3-\sqrt{4F+5}}{2}. \tag{11}$$

From (8) and (11) we see there is a perfect symmetry between the profitability (or not) of a merger involving only leaders and a merger involving only followers: the same proportion of firms is required in each case. From (4) we know the output of leaders is independent of F, hence the only effect of the merger by followers is to induce an expansion of the output of outsiders.

The result of this section can be summarized as follows.

Proposition 1. *In a Stackelberg market, a merger between N_L leaders is profitable iff $N_L>N_L^c$. A merger between N_F followers is profitable iff $N_F>N_F^c$. Moreover, $N_L^c = f(L), N_F^c = f(F), N^c = f(N)$, where N^c is the critical threshold for profitability of a merger in a Cournot market, given by*

$$N^c = f(N) = \frac{2N + 3 - \sqrt{4N + 5}}{2}. \quad (12)$$

N is the number of firms in the Cournot market, and f is the common functional form defining the critical number of firms.[3]

This result indicates that if followers (leaders) do not participate to a merger, they have no effect on the profitability of a merger involving only leaders (followers). A similar result was derived by Hamada and Takarada (2007). It constitutes a generalization of the results of Huck et al. (2001) and Kabiraj and Mukherjee (2003) who show that two leaders (followers) have an incentive to merge iff there are no other leaders (followers). The results are different, however, when mergers between leaders and followers are allowed, as we will now see.

3. Mergers involving both leaders and followers

3.1 General market structures

We now consider the more general case where a merger involves both leaders and followers, which constitutes the focus of this paper. It is assumed at least one leader and one follower participate in those mergers. Moreover, the merged entity becomes a leader. Thus, with $N_L \in [1, L]$ leaders and $N_F \in [1, F]$ participating to the merger, the number of leaders after the merger is L-N_{L+1}+1, while the number of followers is F-N_F. Such a merger reduces the number of followers more than it reduces the number of leaders, since followers "disappear". From (5) the profits after such a merger are given by

$$\pi_L(L - N_L + 1, F - N_F) = \frac{1}{(L - N_L + 2)^2(F - N_F + 1)}, \qquad \pi_F(L - N_L + 1, F - N_F) = \frac{1}{(L - N_L + 2)^2(F - N_F + 1)^2}. \quad (13)$$

This merger is profitable iff

$$\pi_L(L - N_L + 1, F - N_F) > N_L\pi_L(L, F) + N_F\pi_F(L, F). \quad (14)$$

There is a locus of critical values (N_L^c, N_F^c) which determine whether the merger is profitable or not. Much of the analysis that follows aims at characterizing this locus.

Consider first how the participation of a follower to a merger affects its profitability.

Proposition 2. *If, for a given N_L, a merger is profitable when N_F followers participate in it, then it is also profitable when N_F+1 followers participate. If, for a given N_L, a merger is unprofitable when N_F followers participate in it, then it is also unprofitable when N_F-1 followers participate.*[4]

[3] See Salant et al. (1983), footnote 6.
[4] All proofs are in the appendix.

The gain to the merging entity of adding one follower exceeds the profit that was made by that follower prior to the merger. In a sense, followers benefit the merging leaders by allowing themselves to be "absorbed" by the merged entity. From (4) we know the output of a leader does not depend on the number of followers. By bringing one more follower into the merger, the output expansion of followers is reduced (as there are less followers to expand their output following the merger), while the output expansion of each leader outsider is unchanged. Consequently total expansion of output by outsiders is reduced, the price is higher, and the profits of the post-merger entity are higher. And since the profit that was made by the follower was initially small, the gain its participation provides is always sufficient to compensate for that lost profit.

We now consider the role of leaders. We start by looking at mergers involving one leader and any number of followers. This type of merger is common, since it is easier for a leader to acquire a small follower than to acquire a leader of equal size. Huck et al. (2001) and Kabiraj and Mukherjee (2003) find that a merger between a leader and a follower is always profitable. Proposition 3 generalizes this result.

Proposition 3. *A merger between one leader and any number of followers is always profitable.*

This result changes the received wisdom about merger profitability. With more than one leader present pre-merger, the merger involving one leader and any number of followers does not satisfy the standard threshold for profitability: the participating leader represents less than 80% of leaders. And yet, such a merger is profitable. This is due to the participation of the follower(s) to the merger.

We now turn to mergers involving two leaders and any number of followers. Such mergers are also important, both theoretically and empirically. Theoretically, because one of the often cited results of the merger paradox literature is that bilateral mergers are unprofitable whenever there are outsiders. And empirically, since bilateral mergers between large firms are common, and these firms may want to further monopolize the market by predating on smaller firms.

Proposition 4. *For a merger involving two leaders to be profitable, it must involve at least* N_F^{c2} *followers, with*

$$N_F^{c2} = \frac{(F+1)(k-L)}{2L}, \tag{15}$$

where $k \equiv \sqrt{5L^2 - 8L - 4}$.

N_F^{c2} *increases linearly with F, and increases at a decreasing rate with L. The proportion of followers required for such a merger to be profitable,* $\frac{N_F^{c2}}{F}$, *decreases with F and increases with L.*

If L=2, the condition is automatically satisfied since in this case $N_F^{c2}<0$, and the merger is profitable. However, even if $L>2$, the merger will be profitable if enough followers join in. The participation of followers, as explained above, does not change the output of leaders but increases the market price, making the merger more profitable. The critical number of followers required varies with the market structure. It increases with F, for the more there are followers, the more there are outsiders who will expand their output after the merger, and the more of those outsiders must be absorbed by the merger for it to be profitable. But the proportion of followers required, $\frac{N_F^{C2}}{F}$, decreases with F, indicating that when the number of followers is very large, a lower proportion of them is required for profitability. N_F^{C2} also increases with L, because the larger the number of leaders outside the merger, the larger is their output expansion, and the more followers are needed to participate to compensate for that effect. The proportion of followers increases with L as N_F^{C2} increases with L. Hence, while the presence of more followers in the market increases the scope of profitable mergers, it also implies a larger number of them (but a smaller proportion) need to join to make the merger profitable. Followers participating in the merger are a plus for the merging firms, but followers remaining outside the merger reduce the profitability of the merger.

The acclaimed result that in Cournot markets bilateral mergers are unprofitable, and even in a Stackelberg model, two leaders (followers) have an incentive to merge iff there are no other leaders (followers) (Huck et al., 2001; Kabiraj and Mukherjee, 2003) relies heavily on the assumption that no follower is part of the merger.

Before characterizing how in general the range of profitable mergers looks, it is necessary to establish how a profitable Cournot merger (or, equivalently, a merger that would be profitable in a Stackelberg model, but in which no followers participate) is affected by the participation of followers. The following corollary, which follows from the results already derived, answers this question.

Corollary 1. *A merger involving $N_L \geq N_L^C$ leaders is profitable irrespective of the number of followers participating in the merger.*

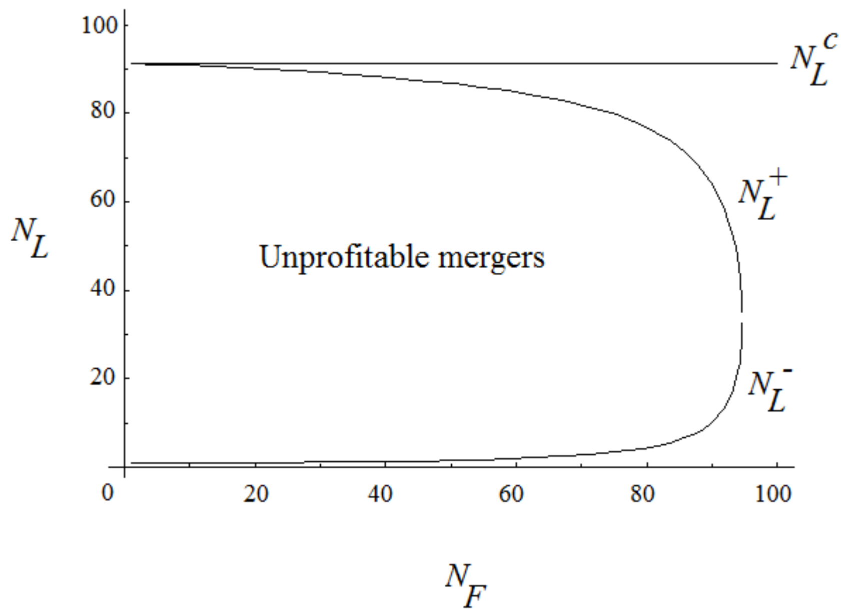
The results derived until now indicate followers have a direct bearing on the profitability of mergers involving leaders and followers. Followers are a desirable target for any solo leader: they make it more likely a merger involving two leaders is profitable, and they enhance the profitability of already profitable mergers.

We are now ready to characterize the profitability of mergers with an arbitrary number of leaders, in particular with $N_L \in [2, N_L^C]$.

Proposition 5. *Unprofitable mergers are in the range $N_L \in [N_L^-, N_L^+]$, with $N_L^- > 1$ and $N_L^+ < N_L^C$; that is, mergers are unprofitable for intermediate values of N_L. Moreover, N_L^- is nondecreasing and N_L^+ is nonincreasing in N_F.*

Figure 1 illustrates how the range of profitable mergers varies with N_L and N_F. Consider a given value of N_F. When $N_L < N_L^-$ or $N_L > N_L^+$, the merger is profitable. When, however, $N_L \in [N_L^-, N_L^+]$, the merger is unprofitable. The intuition for this result is that the profits of leaders are initially high, therefore a substantial degree of market power of the post-merger firm is required to cover their lost profits. When the number of leaders involved is intermediate and the number of followers is small, the market power of the post-merger firm is insufficient to compensate for the lost profits, and the merger is not profitable.

Figure 1. Number of participating leaders and merger profitability (*L*=*F*=100)



The result that N_L^- is nondecreasing and N_L^+ is nonincreasing in N_F (see the last claim of proposition 5 and figure 1) means the participation of more followers to the merger reduces the number of leaders required for profitability. Moreover, the participation of followers reduces the number of leaders required for profitability compared with the case where only leaders merge. Similarly, the participation of leaders reduces the number of followers required for profitability compared with the case where only followers merge.

This result reverses one of the main results of the merger paradox literature, "if a merger by a specified number of firms causes losses (respectively, gains), a merger by a smaller (larger) number of firms will cause losses (gains)" (Salant et al., 1983:193), so the solution to merger unprofitability was to enlarge the pool of participating firms. In a Stackelberg model, a merger may become unprofitable by adding more leaders, and may become profitable by removing leaders.

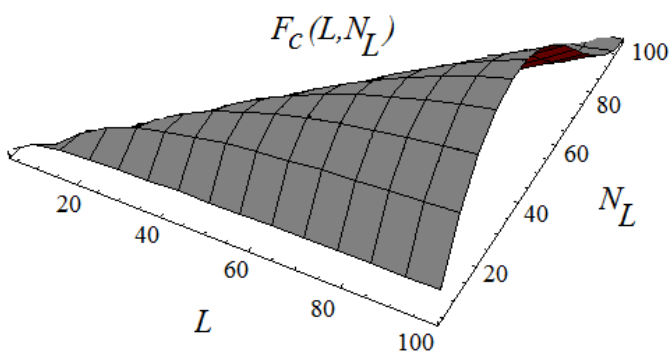
Until now we concentrated on the number of leaders required for profitability, noting that followers facilitate and enhance profitability. We now take a closer look at the market side of followers. In fact, for a certain number of followers to join a merger and make it profitable, there must be enough followers in the first place. At the same time, as explained in proposition 4, the number of followers required to make a merger involving two leaders (this will also be true for any N_L) profitable increases with F.

The following two propositions clarify the role of F in merger profitability. Consider the profitability of mergers involving all followers. Is this sufficient to ensure profitability? The following proposition gives a negative answer to this question.

Proposition 6. *There exists a critical number of followers that must be present prior to the merger, F^c, such that, for a merger involving N_L leaders and all followers (N_F=F), the merger is profitable iff $F>F^c$. Moreover, F^c increases with L, and increases and then decreases with N_L.*

Proposition 6 takes the extreme case of all followers joining the merger and asks the question: is this sufficient to ensure profitability? The answer depends on L and N_L. The critical threshold F^c indicates the minimal number of followers such that, if all followers joint the merger, it will become profitable. This threshold increases with L: as the number of leaders becomes large relative to the number of followers, the likelihood increases that even if all followers join, the merger will not be profitable (for a given N_L). Thus, a small number of followers reduces the scope of benefitting from them to ensure merger profitability. Also, F^c increases and then decreases with N_L. As explained above, mergers are more likely to be profitable when a small or a large number of leaders join in. When N_L is small, the merger is more likely to be profitable, and less followers are needed (assuming they all participate in the merger) to make the merger profitable. As N_L increases, however, the merger becomes unprofitable, and more followers are needed to join the merger. Beyond a certain point, however, the joining leaders have less and less market power prior to the merger, and they require a lesser increase in profits by the merged entity, hence the need for followers declines, and F^c declines with N_L. Figure 2 illustrates F_c as a function of L and N_L.

Figure 2. Number of followers required in the market when $N_F=F$ (L=100)



The result that F^c increases and then decreases with N_L (from proposition 6) is another facet of the result that when mergers are unprofitable, they are so for intermediate numbers of leaders participating in the merger (from proposition 5). F^c reaches its maximum for intermediate values of N_L, which means it is for those values of N_L that the most followers are needed to make the merger profitable. Whereas, F^c is smaller at small and large values of N_L, meaning that for those values of N_L only a few followers are needed to make the merger profitable.

As for mergers involving only a portion of followers, figure 1 also gives, indirectly, the number of followers required for a merger to be profitable for a given N_L. N_L^- and N_L^+ define a function, N_F^c such that a merger is profitable iff $N_F>N_F^c$. This function has an inverted-U shape. For N_L very small, the merger is profitable with any strictly positive number of followers. For N_L very large, in particular for $N_L>N_L^c$, the merger is profitable even without any followers. In between, N_F^c first increases then decreases with N_L. It is easy to verify that the number of followers required for profitability, N_F^c, follows the same pattern as F^c: it increases linearly with F, and increases and then decreases with N_L.

The following corollary takes this argument one step further, and asks the question: suppose all followers join the merger; when are enough of them around for a merger to become profitable irrespective of the number of leaders joining in (but at least one, to induce leadership)?

Corollary 2. *If $F>F^{cc}=\max F^c$, then a merger involving all followers is profitable for any number of leaders participating in the merger. Moreover, F^{cc} is increasing in L.*

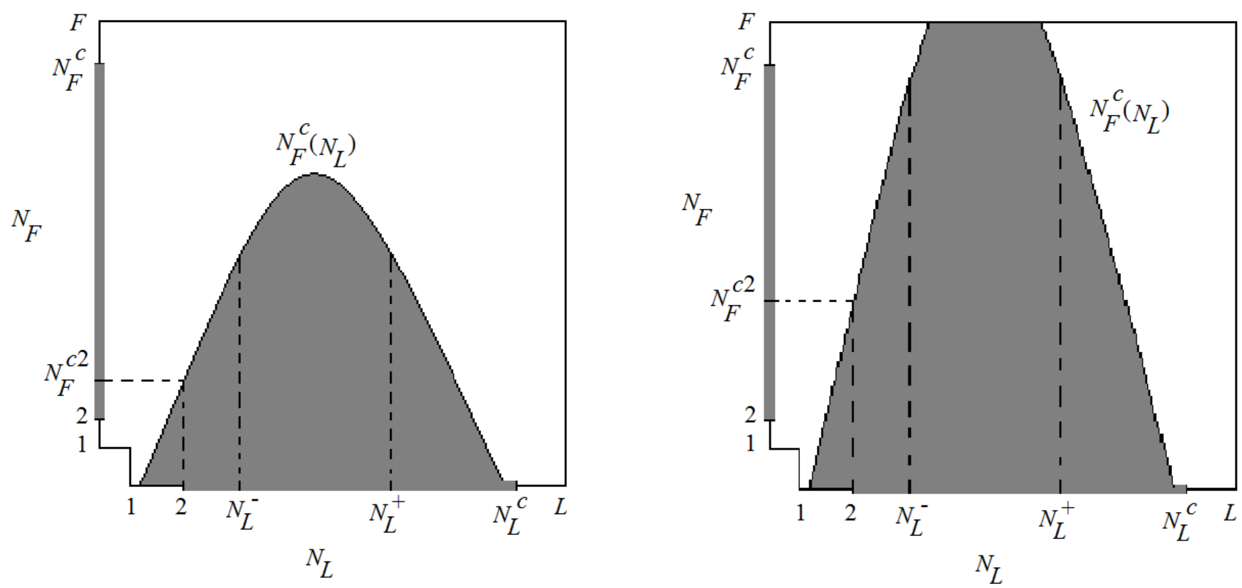
This corollary says that if F is large enough relative to L, then a merger where all followers join will be profitable, irrespective of N_L. The critical value of F required for this to occur is, for a given L, the maximum number of followers that was required by a specific number of leading firms. If F is large enough for the merger to be profitable with this value of N_L, then it will also be large enough for the merger to be profitable for any other value of N_L. This is why F^{cc} is obtained by maximizing F^c over N_L: if the merger is profitable under harsh conditions (N_L is such that a very large number of followers are needed), then it will be profitable under easier conditions (N_L requires less followers to join the merger to make it profitable).

Followers provide a double benefit from the point of view of merger profitability for leaders. On the one hand, the standard threshold is unchanged from a Cournot model, hence even if they don't join a merger they do not reduce the likelihood of its profitability. On the other hand, they can increase the likelihood of a merger becoming profitable by participating in it, and acting as "substitutes" for other leaders who haven't joined.

Figure 3 illustrates the regions of profitable and unprofitable mergers, using the different thresholds derived above, in two cases: the case where $F>F^{cc}$, so that a merger involving all followers is profitable for any number of participating leaders (figure 3a), and the case $F<F^{cc}$, so that even participation by all followers does not guarantee profitability (figure 3b). These two figures incorporate also mergers involving only leaders or only followers (on the axes), which must satisfy the Cournot threshold to be profitable.

Figure 3. Profitable and unprofitable mergers

(a) $F>F^{cc}(L)$ (b) $F<F^{cc}(L)$



Note: Grey areas represent unprofitable mergers.

3.2 Specific market structures

Now that we have analyzed some general properties of mergers in Stackelberg models, we turn to some specific market structures to obtain more specific results. Three symmetric and two asymmetric market structures are considered: (L=2,F=2), (L=3,F=3), (L=10,F=10), (L=10,F=3) and (L=3,F=10). These choices represent sufficient variety in terms of degree of concentration and asymmetry between leaders and followers to illustrate the results derived above.

Consider first the market structures (L=2,F=2) and (L=3,F=3). In the (L=2,F=2) market, there are four possible mergers (with at least one leader and one follower): (N_L=1,N_F=1), (N_L=1,N_F=2), (N_L=2,N_F=1), and (N_L=2,N_F=2). In the (L=3,F=3) market, there are 9 possible mergers.

Proposition 7. *With* (L=2,F=2) *or* (L=3,F=3), *all mergers involving at least one leader and one follower, with the merged entity acting as a leader, are profitable.*

In the (L=2,F=2) market, all possible mergers are profitable. Except for the merger (N_L=2,N_F=2), all other three mergers entail only 50% of leaders and/or 50% of followers are engaged in the merger. This is well below the 80% threshold found when only leaders or only followers merge together (see proposition 1); yet, these merges are profitable. They are profitable because they entail the participation of firms with different market positions. Take the merger (N_L=2,N_F=1). This merger is profitable even though it involves only one follower; by incorporating the two leaders into the merger, the total output expansion by outsiders is reduced, and the remaining follower cannot hurt the merging entity.

The other market structure considered in proposition 7 is (L=3,F=3). There are 9 possible mergers, all of them profitable. Take for instance the merger (N_L=2,N_F=1). This merger is between firms which have a market share well below what is required when only leaders are involved. Yet, it is profitable because one follower joined in. The joining of the follower does not change the output of the merging entity (see equation 4), but increases the price sufficiently (since that follower disappears) to make the merger profitable in spite of the output expansion by outsiders. In a sense, the follower acts as a "substitute" for the third leader who is not part of this merger.

Consider now the market structure (L=10,F=10). Based on (14), a merger of N_L and N_F firms is profitable iff

$$\frac{1}{(11-N_F)(12-N_L)^2}-\frac{11N_L+N_F}{14641}>0. \tag{16}$$

There are 100 possible mergers in this market. Figure 4 illustrates the possible mergers and their profitability, based on (14). When $N_F \leq 6$, a merger is profitable iff N_L is very small or very large; when $N_F > 6$, all mergers involving at least one leader are profitable. This is consistent with proposition 5, which establishes that mergers can be unprofitable for intermediate numbers of participating leaders.

To make more precise the profit comparisons behind figure 4, consider the row of figure 4 with N_F=6. This row indicates that a merger involving 6 followers is profitable iff $N_L \in \{1,2,6,7,8,9,10\}$. With L=F=10 and N_F=6, a merger involving N_L leaders is profitable iff

$$\pi_L(10-N_L+1,10-6)-N_L\pi_L(10,10)-6\pi_F(10,10)=\frac{1}{5(12-N_L)^2}-\frac{11N_L+6}{14641}>0. \tag{17}$$

This expression has two roots: N_L=2.3, and N_L=5.2. Hence, the merger is not profitable for $N_L \in [3,5]$ and is profitable otherwise.

To better understand this result, figure 5 illustrates the pre-merger and post-merger profits as a function of N_L with N_F=6. When N_L=1, the merger is highly profitable; when N_L=2, the merger is still profitable, but only marginally. When the second leader joins in, its contribution to the profits of the merged firms is less than the profits sacrificed when that firm was not merged (this is reflected by the slope of the curve, which is lower than the slope of the straight line at that point). However, since the merger profits were so high at N_L=1, the resulting merger profits at N_L=2 are still higher than with no merger. However, as the third and subsequent leaders join, the merger is not profitable anymore. Only when N_L=6 does the merger become profitable again. Thus, there is a discontinuity in the relationship between the number of leaders joining the merger and its profitability.

Figure 4. Merger profitability with L=F=10

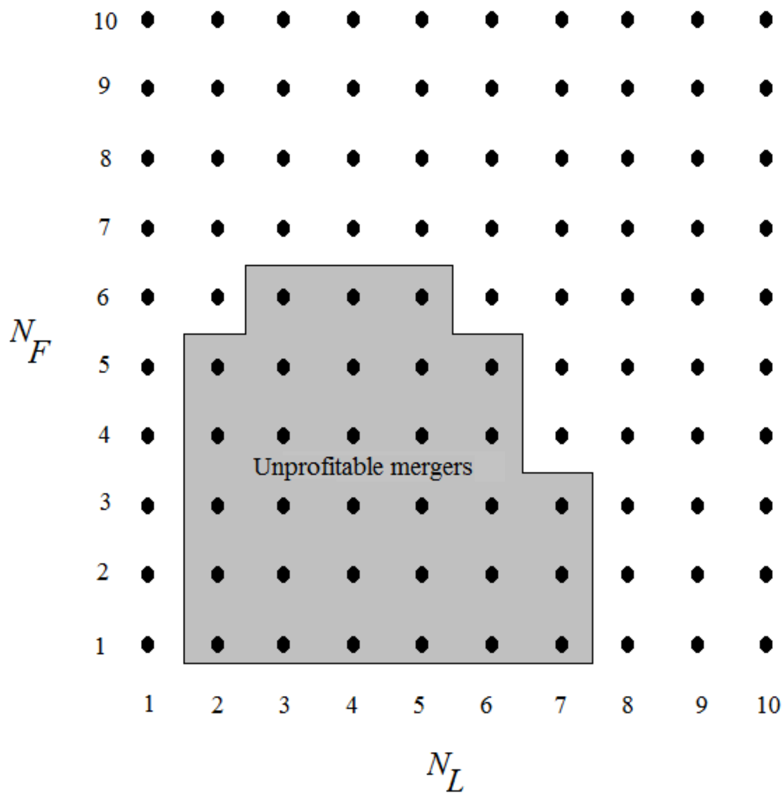
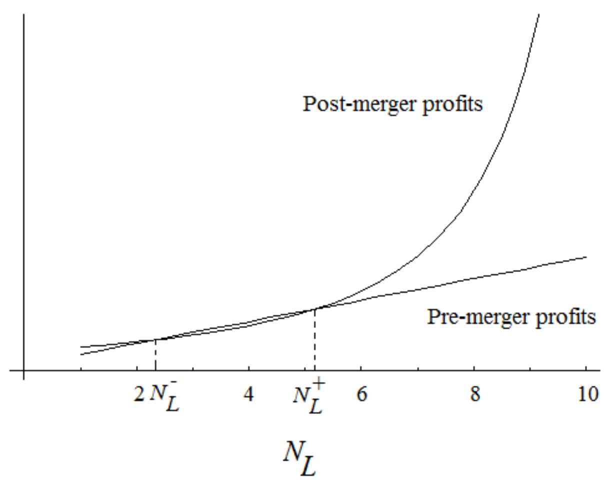
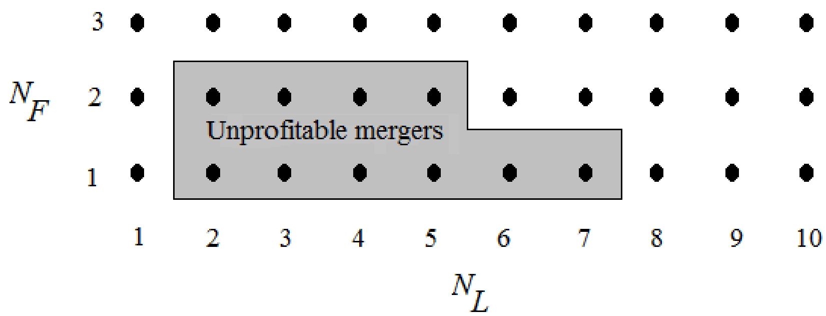


Figure 5. Pre and post-merger profits (L=F=10, N_F=6)



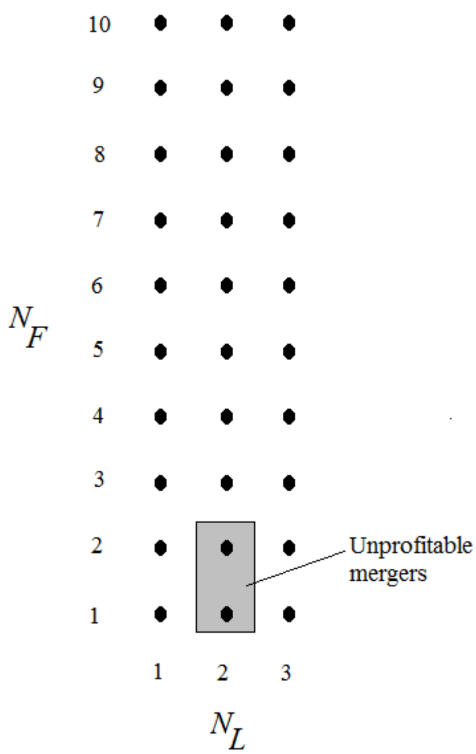
Consider now the asymmetric market structure (L=10,F=3). Figure 6 illustrates the results. The range of profitable mergers is qualitatively similar to the range obtained for the previous market structure (L=10,F=10). With (L=10,F=3), if all followers participate, then the merger is profitable for any number of participating leaders; if at least one follower does not participate, then the merger is profitable iff N_L is very small (N_L=1) or relatively large, again with a substitution between leaders and followers: the more followers participate, the less leaders are required to do so.

Figure 6. Merger profitability with L=10, F=3



The last market structure considered, (L=3,F=10), yields a similar result. As figure 7 shows, the only unprofitable mergers are those with (N_L=2,N_F≤2). Again, an intermediate number of leaders and a small number of followers yield an unprofitable merger. Given the small number of leaders, there is no scope for substitution between leaders and followers here.

Figure 7. Merger profitability with L=3, F=10



The comparison between figures 6 and 7 reveals the range of unprofitable mergers is larger when L is large relative to F than when the converse is true. This is because a large L expands the range of "intermediate" values of N_L, for which mergers are less likely to be profitable. Therefore, we can expect more merger unprofitability in markets where the number of leaders is large relative to the number of followers.

The study of these specific market structures illustrates the results derived above for more general market structures. For example, figures 4, 6 and 7 are illustrations of the case depicted in figure 3a, where the participation of all followers guarantees merger profitability irrespective of N_L.

3.3 Welfare

What is the impact of mergers involving at least one leader and one follower on welfare? Such mergers increase the degree of concentration and, not surprisingly, are welfare reducing. First note that any merger increases price. To see that, we substitute equilibrium outputs from (4) into the demand function to get the equilibrium price:

$$\begin{aligned} P^*(L,F) &= 1 - Ly_L^*(L) - Fy_F^*(L,F) \\ &= 1 - L\frac{1}{L+1} - F\frac{1}{(L+1)(F+1)} \\ &= \frac{1}{(L+1)(F+1)}. \end{aligned} \tag{18}$$

It is obvious from (18) that price decreases with the number of firms, hence any merger will raise price and hurt consumers.

Define consumer surplus as

$$CS(L,F) = \frac{Y(L,F)^2}{2}. \tag{19}$$

Define welfare as a function of the number of leaders and followers as

$$W(L,F) = CS + L\pi_L + F\pi_F. \tag{20}$$

The next proposition formalizes the impact of merger on consumers, outsiders, and total welfare.

Proposition 8. *Any merger involving at least one leader and one follower, with the merged entity acting as a leader, reduces consumer surplus and welfare, and increases total profits and the profits of each outsider.*

Because the merger entails an increase in concentration and firms are symmetric, even though it may or may not benefit the merging firms, it always benefits outsiders, in addition to reducing consumer surplus (i.e. increasing price) and welfare.[5] Thus, the vigilance of antitrust authorities need not be relaxed because a merger involves a mixture of leader and follower firms.

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[5] Experimental evidence suggests mergers in Stackelberg models benefit outsiders and reduce consumer surplus, as predicted by theory, buy they do not change the profits of the merging firms, contrary to theoretical predictions. See Huck (2005).

4. Conclusions

This paper has extended the analysis of the profitability of mergers to Stackelberg models. Followers alleviate the merger paradox, in that they increase the range of profitable mergers for leaders. Unprofitable mergers are typically those involving an intermediate number of leaders and a small number of followers. All mergers involving at least one leader and one follower are welfare reducing. Note that it is the participation of followers to the merger, rather than their presence per se, which makes mergers profitable more often. Hence, it is not really Stackelberg leadership, but the absorption of followers, which enhances merger profitability.

Followers are typically smaller in size than leaders. When merger between a number of leaders is unprofitable, it may be easier to bring in one or more followers than to bring in additional leaders, as followers are smaller in size. Yet, in spite of their small size, they may be sufficient to make the merger profitable. The model can also be interpreted as saying that small firms, in spite of their small size, can contribute to making mergers between larger firms profitable.

Appendix

Proof of proposition 2.

$$\frac{\partial \pi_L(L-N_L+1,F-N_F)}{\partial N_F}=\frac{1}{(L-N_L+2)^2(F-N_F+1)^2}>\frac{\partial[N_L\pi_L(L,F)+N_F\pi_F(L,F)]}{\partial N_F}=\frac{1}{(L+1)^2(F+1)^2}\,.\blacksquare \quad \text{(A1)}$$

Proof of proposition 3. Evaluating (14) at N_L=1 yields

$$\pi_L(L-1+1,F-N_F)-\pi_L(L,F)-N_F\pi_F(L,F)=\frac{N_F^2}{(L+1)^2(F+1)^2(F+1-N_F)}>0.\blacksquare \quad \text{(A2)}$$

Proof of proposition 4. Substituting N_L=2 into (14), setting it equal to zero, and solving for N_F yields (15).

$$\frac{\partial N_F^{c2}}{\partial F}=\frac{k-L}{2L}>0. \quad \text{(A3)}$$

This last derivative is independent of F, implying that N_F^{c2} increases linearly with F.

$$\frac{\partial N_F^{c2}}{\partial L}=\frac{2(L+1)(F+1)}{kL^2}>0. \quad \text{(A4)}$$

$$\frac{\partial^2 N_F^{c2}}{\partial L^2}=-\frac{2(F+1)(10L^3+3L^2-24L-8)}{(kL)^3}<0. \quad \text{(A5)}$$

$$\frac{\partial \frac{N_F^{c2}}{F}}{\partial F}=\frac{L-k}{2LF^2}<0. \quad \text{(A6)}$$

Given that N_F^{c2} is increasing in L, the proportion $\frac{N_F^{c2}}{F}$ is also increasing in L. ■

Proof of corollary 1. From proposition 1 we know that a merger involving leaders only is profitable iff $N_L \geq N_L^c$. And from proposition 2 we know that adding a follower to a merger increases its profitability. ■

Proof of proposition 5. The first claim follows from propositions 1 and 3. Proposition 3 establishes that $N_L^- > 1$, while proposition 1 establishes that $N_L^+ < N_L^c$.

The effect of N_F on N_L^- and N_L^+ is hard to establish without fixing L and F. For now we fix L and F and illustrate the result. From section 3.2 it is clear that this result is valid for more general market structures. Let $L=F=100$. For a given value of N_F, solving (14) for the critical value(s) of N_L such that the merger is just profitable yields two roots. The first of these roots is what we call N_L^-; the second root is N_L^+; figure 1 plots both curves. N_L^- is nondecreasing and N_L^+ is nonincreasing in N_F. On this figure is plotted also N_L^c, which is an upper bound to N_L^+. ■

Proof of proposition 6. Setting $N_F=F$ in (14) we can solve for F^c:

$$F^c(L,N_L) = \frac{N_L^3 - N_L^2(2L+3) + N_L L(L+2) - L^2 + 2 + (L - N_L + 2)\sqrt{(N_L - 1)[L^2(N_L+3) - 2L(N_L^2 + N_L - 2) + N_L(N_L^2 - N_L - 4)]}}{2(L+1)^2} \quad \text{(A7)}$$

This function depends in a complex way on L and N_L. It is illustrated in figure 2. It increases steadily with L, and increases and then decreases with N_L. ■

Proof of corollary 2. This follows from proposition 6. And F^{cc} is increasing in L since F^c is increasing in L. ■

Proof of proposition 7. Substituting $L=2$ and $F=2$ into (14) yields

$$\frac{1}{(3-N_F)(4-N_L)^2} - \frac{3N_L + N_F}{81} > 0. \quad \text{(A8)}$$

This expression is positive for all combinations of $N_L \in \{1,2\}$ and $N_F \in \{1,2\}$.

Substituting $L=3$ and $F=3$ into (14) yields

$$\frac{1}{(4-N_F)(5-N_L)^2} - \frac{4N_L + N_F}{256} > 0. \quad \text{(A9)}$$

This expression is positive for all combinations of $N_L \in \{1,2,3\}$ and $N_F \in \{1,2,3\}$. ■

Proof of proposition 8. The change in consumer surplus is given by

$$CS(L - N_L + 1, F - N_F) - CS(L,F) = \frac{1}{2}\left[\left(\frac{L - N_L + 1 + (F - N_F)(L - N_L + 2)}{(L - N_L + 2)(F - N_F + 1)}\right)^2 - \left(\frac{L + F + LF}{(L+1)(F+1)}\right)^2\right] < 0. \quad \text{(A10)}$$

This expression is negative for all $L,F>0$, $N_L \in [1,L]$, $N_F \in [1,F]$.

The change in the profit of an outsider leader (if any) is given by

$$\pi_L(L - N_L + 1, F - N_F) - \pi_L(L,F) = \frac{1}{(L - N_L + 2)^2(F - N_F + 1)} - \frac{1}{(L+1)^2(F+1)} > 0. \quad \text{(A11)}$$

This expression is positive for all $L,F>0$, $N_L \in [1, L-1]$, $N_F \in [1, F]$.

The change in the profits of a follower are

$$\pi_F(L-N_L+1, F-N_F) - \pi_F(L,F) = \frac{1}{(L-N_L+2)^2(F-N_F+1)^2} - \frac{1}{(L+1)^2(F+1)^2} > 0. \quad \text{(A12)}$$

This expression is positive for all $L,F>0$, $N_L \in [1, L]$, $N_F \in [1, F-1]$.

The change in total profits is given by

$$\begin{aligned}&(L-N_L+1)\pi_L(L-N_L+1, F-N_F) + (F-N_F)\pi_F(L-N_L+1, F-N_F) - L\pi_L(L,F) - F\pi_F(L,F) = \\ &\frac{L-N_L+1}{(L-N_L+2)^2(F-N_F+1)} + \frac{F-N_F}{(L-N_L+2)^2(F-N_F+1)^2} - \frac{L}{(L+1)^2(F+1)} - \frac{F}{(L+1)^2(F+1)^2} > 0.\end{aligned} \quad \text{(A13)}$$

This expression is positive for all $L,F>0$, $N_L \in [1, L]$, $N_F \in [1, F]$.

The change in welfare is given by

$$\begin{aligned}W(L-N_L+1, F-N_F) - W(L,F) = {} & CS(L-N_L+1, F-N_F) - CS(L,F) \\ & + (L-N_L+1)\pi_L(L-N_L+1, F-N_F) - L\pi_L(L,F) \\ & + (F-N_F)\pi_F(L-N_L+1, F-N_F) - F\pi_F(L,F) < 0.\end{aligned} \quad \text{(A14)}$$

Substituting consumer surplus and profits, it is easy to verify that (A14) is negative for all $N_L \in [1, L]$, $N_F \in [1, F]$. ■

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