CAHIER DE RECHERCHE #1407E
Département de science économique
Faculté des sciences sociales
Université d'Ottawa

WORKING PAPER #1407E
Department of Economics
Faculty of Social Sciences
University of Ottawa

Robust Wagstaff Orderings of Distributions of Self-Reported Health Status

Paul Makdissi* and Myra Yazbeck†

September 2014

* Department of Economics, University of Ottawa, 55 Laurier E. (10125), Ottawa, Ontario, Canada, K1N 6N5; Email: paul.makdissi@uottawa.ca

† School of Economics, 612- Colin Clark (39) Building, University of Queensland, Brisbane, QLD 4072, Australia; Email: m.yazbeck@uq.edu.au

Abstract

When assessing socioeconomic health inequalities researchers often draw upon measures of income inequality that were developed for ratio scale variables. As a result, the use of categorical data (such as self-reported health status) produces rankings that may be arbitrary and contingent to the scaling adopted. In this paper, we develop a method that overcomes this problem by providing conditions for which these rankings are invariant to the scaling function chosen by the researcher. In doing so, we draw on the insight provided by Alkire and Foster (2004) and extend their method to the dimension of socioeconomic inequality exploiting the properties of Wagstaff's class of indices. We then provide an empirical illustration using the National Institute of Health Survey 2012.

Keywords: *Health Achievement, Health Inequality, Stochastic Dominance, Self-Reported Health Status.*
JEL classification: D63, I14.

Résumé

Lorsqu'ils analysent les inégalités socioéconomiques de santé, les chercheurs utilisent souvent des indices qui ont été développés pour mesurer les inégalités de revenus, un variable d'échelle de ratio. Il en résulte que les classements générés peuvent être arbitraires et contingent à une échelle numérique particulière lorsqu'on analyse des distributions de variables de santé catégoriques telles que l'auto-évaluation de l'état de santé. Dans cet article, nous proposons une méthode qui permet de résoudre ce problème en identifiant les conditions pour lesquelles ces classements sont invariants aux échelles numériques choisies par le chercheur. Pour ce faire, nous construisons sur l'idée d'Allison et Foster (2004) et étendons leur méthode afin de prendre en compte la dimension socioéconomique des inégalités de santé dans le cadre de la classe d'indices de Wagstaff. Nous illustrons empiriquement la méthode à l'aide de données du National Institute of Health Survey 2012.

Mots clés : *Niveau de santé, inégalités de santé, dominance stochastique, auto-évaluation de l'état de santé*

Classification JEL : D63, I14.

1 Introduction

Health inequality is, and has been for long, considered to be an important concern for policy makers that raised many challenges. While continuous efforts were made to decrease socioeconomic health inequalities through different policy interventions, the monitoring and evaluation of health policies and reforms still relies on measures borrowed from income inequality literature. Beyond any doubt, these methods are valuable. This being said, one cannot deny that fundamental differences between the type of variables used in income inequality analysis and the type of variables utilized when conducting health inequality analysis raise several methodological challenges that may produce misleading conclusions. It is widely recognized that a large body of the health inequality measurement literature is based on tools developed for situation where variables are cardinal and where the value of 0 has a well defined meaning (e.g., income, expenditures, weight or height). The reality is that health researchers often use population surveys where most of the available information on health status is given in the form of categorical variables. As a result, one cannot directly provide a reliable measure of inequalities in health status by computing well-known income inequality indices. Previous attempts to address this problem offered different methods, yet the solutions provided are far from being complete. While a pioneer work by Allison and Foster (2004) offered a solution (using a stochastic dominance approach) at the cost of forgoing the socioeconomic dimension of health inequalities, a subsequent work by Zheng (2011) addressed this caveat at the cost of overlooking heterogeneity within socioeconomic classes. Lately, Makdissi and Yazbeck (2014) provided a solution that captures the socioeconomic dimension of health inequalities however this method is only applicable in the context of multiple categorical data and forgoes the depth of health problems.

This paper is motivated by the importance of the role of measures of socioeconomic health inequalities in providing information that can help shaping health policies. Its objective is to fill an existing gap in the literature by proposing a method that allows for robust orderings of health achievement and health inequality for self-reported health status's distributions.[1] In doing so, it develops a new tool for researchers who wish to use categorical variables and capture (simultaneously) health inequality and the socioeconomic dimension without forgoing the depth of the health problem or the heterogeneity within socioeconomic classes. To illustrate the proposed methodology we use the National Institute of Health Survey public use data for 2012. We find partial orderings that suggest that the South is generally dominated by the Northeast, the Midwest and the West as far as health achievements are concerned. When socioeconomic health inequalities are considered, clear and robust patterns are more difficult to obtain. These empirical results caution us against imposing numerical scales when dealing with ordinal data, as they may provide full rankings that are not necessarily robust.

The remaining of this paper is organized as follows. Section 2 presents a review of the related literature. Section 3 lays out the theoretical framework. Section 4 presents the data used for the empirical illustration. Section 5 present the results of the empirical illustration. Finally section 6 concludes.

2 Literature Review

While the concentration index is nowadays a widely accepted measure of socioeconomic health inequality, it is important to portray its measurement limitations. First, concentration indices do not account for the average level of health in the population considered (Wagstaff, 2002). Thus, a policy that improves the average level of health, while keeping the relative distribution of health constant, will be deemed neutral when using the concentration index. To overcome this problem, Wagstaff (2002) proposes the use of an achievement index that captures simultaneously the average level of health status and the socioeconomic inequality of its distribution. Second, the concentration index fails to provide consistency between the rankings of health attainments and health shortfalls:

[1] In this paper Wagstaff orderings refers to the ordering provided by Wagstaff's class of indices as defined in Wagstaff (2002).

this is referred to as the mirror problem (Clarke *et al.*, 2002). Erreygers (2009) suggested a corrected version of the concentration index that accounts for this inconsistency and highlights that this index is not an index of relative inequality.[2]

The last measurement problem, the focal measurement issue of this paper, is the *arbitrariness of the concentration index* (for details see Erreygers, 2006 and Zheng, 2008). It results from a common misuse of non-ratio-scale variables while computing inequality indices that are developed for ratio-scale variables (for details see Erreygers and Van Ourti 2011, Makdissi and Yazbeck 2014). There is a rapidly growing body of literature that addresses *the arbitrariness of the concentration index*, most of it is relatively recent. In a seminal work, Allison and Foster (2004) offer a solution in the dimension of pure health inequality. They propose a stochastic dominance approach to identify robust rankings of health distributions. When considering two distributions, they show that if the cumulative proportion of individuals with a self-reported health status below a specific threshold is lower in a distribution for all possible thresholds, then the average of self-reported health status will be higher for this distribution for all possible numerical scale. They also develop a similar test to rank distribution according to their dispersion (inequality). Thus, by using this approach the researcher can account for the depth of health status but at the cost of overlooking the socioeconomic dimension of health inequality. Building on Allison and Foster (2004), Abul Naga and Yalcin (2008) develop a class of health inequality indices that is readily applicable to categorical health variables. These indices are based on the proportion of the population that falls within each health categories rather than on the cardinal (meaningless) value of the health statuses. As in Allison and Foster (2004), the authors acknowledge that the mean of a health variable has no meaningful interpretation for categorical variables. Consequently, in their axiomatization of the proposed indices, they redefine aversion to inequality as aversion to *median preserving spreads*. On the empirical front, Madden (2010) offers an application in which he compares both Allison and

[2]Subsequently, Lambert and Zheng (2011) show that no index of relative inequality can really avoid this problem.

Foster (2004) and Abul Naga and Yalcin (2008) approaches with other methods where a cardinal scale is imposed on categorical health information. The author finds that when ordinal data is used the results are very sensitive to the analytical framework selected.[3] Subsequent work by Lazar and Silber (2013) generalize Abul Naga and Yalcin's work on the measurement of pure health inequality in presence of categorical health variables. They show that the class of Abul Naga and Yalcin (2008) health inequality indices is a special case of a more general class of health inequality indices that can be developed using Reardon's (2009) class of segregation indices. In a recent contribution, Cowell and Flachaire (2012) propose another class of pure health inequality indices for categorical health data. Similarly to Abul Naga and Yalcin (2008), their class is based on the population shares within each categories, and is thus related to the well-known class of generalized entropy income inequality indices.

Whilst a lot of research was targeted towards developing measures of *pure health* inequality for categorical health data, the literature on the measurement of *socioeconomic health inequality* remains very scant. To our best knowledge there are only two papers that revisits this problem without imposing a numerical scale. Zheng (2011) proposes an approach that requires the researcher to group individuals in socioeconomic classes in a first step then impose monotonicity of health in socioeconomic ranks in a second step. While this monotonicity assumption may be satisfied in presence of a limited number of socioeconomic classes, it does not always hold for all individual socioeconomic ranks. Using Zheng's approach, it is possible to use non ratio-scaled variables when analyzing socioeconomic health inequalities however, this comes at the cost of overlooking heterogeneity within socioeconomic classes. Makdissi and Yazbeck (2014) propose a solution to the problem of arbitrariness of the socioeconomics health inequality measure in a context where health status can be assessed using multiple categorical variable. They use a counting approach that focuses on the breadth of the health problem rather than the depth. This allows them to obtain a

[3] It should be emphasized that this paper does not only compare two methods but also two different measures of health inequalities namely pure health inequality and socioeconomic health inequalities.

well defined 0 for the health status variable and thus use Wagstaff's class of health achievement and socioeconomic health inequality indices. While this count approach provides consistent values and rankings, this consistency comes at the cost of overlooking the depth of health problems. Indeed, the literature has not yet offered a complete solution for the *arbitrariness of inequality and achievement indices* in the dimension of socioeconomic health inequality.

This paper is closely related to the work of Allison and Foster (2004) and the work of Abul Naga and Yalcin (2008) yet it diverges from these papers as far as the dimension of health inequality is concerned. While Allison and Foster (2004) and Abul Naga and Yalcin (2008) focus on measurement issues in the dimension of pure health inequality, this paper investigates the measurement issue in the dimension of socioeconomic health inequality. In a first step, it builds on Allison and Foster (2004) and extend their analysis to account for the socioeconomic dimension of health inequality. Thus, instead of identifying robust comparisons of averages of health distributions, we identify robust comparisons of Wagstaff (2002) health achievement indices based on the proportion of population in each health category. A particularity of these indices is their sensitivity to both the average and the socioeconomic inequality of health statuses in the population. In a second step, we propose a new class of health inequality indices that introduces Wagstaff's (2002) socioeconomic weights in the evaluation of the *Average Absolute Deviation about the Median*. Finally, we propose a method to identify robust rankings of these socioeconomically weighted average absolute deviation about the median.

3 Theoretical Framework

This section discusses the measurement problem arising from the use of categorical variables when computing Wagstaff's class of health achievement indices as well socioeconomic health inequality indices. To present a clear picture of the situation, we first overview the measurement framework in which we are operating, then provide a numerical example that highlights the problem concretely. Finally, we offer a solution that allows the researcher to obtain robust Wagstaff orderings of distributions of self-reported health status.

3.1 Wagstaff health achievement indices of self-reported health statuses

Assume that a health researcher would like to analyze socioeconomic health inequalities in a population of N individuals where relevant information on the joint distribution of health and socioeconomic statuses is given by $\{(h_i, r_i)\}_{i=1}^{N}$, where h_i represents health status and r_i the rank in the distribution of living standards (income, total expenditures, occupational categories, education level, etc), starting from the lowest level to the highest level of living standards. Suppose that information on individual health status is available in the form of a categorical variable (e.g., self-reported health status). Without loss of generality, assume that there are K health categories such that $h_i \in \{1, 2, ..., K\}$. We would like to inform the policy maker about the the health system performance in terms of health achievement and of socioeconomic health inequalities.[4] As argued earlier, Wagstaff's achievement index captures simultaneously total health in the distribution and the socioeconomic inequalities in health statuses between the poor and the better-off. This makes this class of indices a natural starting point for analyzing health achievement and socioeconomic health inequalities.

In the above context, the achievement index is given by:

$$A(\nu) = \sum_{i=1}^{N} \omega(r_i; \nu) c(h_i), \tag{1}$$

where

$$\omega(r_i; \nu) = \frac{(N - r_i + 1)^{\nu} - (N - r_i)^{\nu}}{N^{\nu}}, \ \nu \geq 1. \tag{2}$$

Following Yitzhaki (1983), ν can be interpreted as a parameter of aversion to socioeconomic health inequality.[5] If $\nu = 1$, there is no aversion to socioeconomic health inequality and the associated achievement index, $A(1)$ is the average health status, μ_c.[6] If $\nu > 1$, then the achievement index

[4] The value of h_i only indicates the rank of the category from the lowest, 1, to the highest, K,

[5] Note that Yitzhaki (1983) considers the context of income inequality.

[6] It is important to mention that $\omega(r_i; 1) = \frac{1}{N} \forall r_i$.

becomes sensitive to socioeconomic inequalities in health.

To compute a health achievement index, one must first select a scale, $c(h)$ which assigns a numerical value to each category h of health. One possible choice is to use directly the ranks of h. In this case, $c(h) = h$. Alternatively, one may allow differences between adjacent category to vary. In fact, any monotonically increasing $c(h)$ is an admissible numerical scale as it will preserve the same information included in h.

Unfortunately, in the case of categorical variables, such as self-reported health statuses, Wagstaff's achievement indices are sensitive to a re-scaling. Given two populations A and B, the ranking provided by the achievement index may be contingent to the choice of the scaling function $c(h)$. To illustrate this problem, consider two bivariate distributions of socioeconomic status and health (as in Table 1) and three possible scaling functions (as in Table 2). For a scaling function $c_1(h)$, achievement indices for distributions A and B (when $\nu = 2$)[7] are $A_A(2) = 2.80$ and $A_B(2) = 2.95$. These values change to $A_A(2) = 9.20$ and $A_B(2) = 8.95$ when scaling function $c_2(h)$ is used, and $A_A(2) = 3.40$ and $A_B(2) = 3.90$ when $c_3(h)$ is used. Note that health achievement in distribution A is higher than in distribution B when using the scaling functions $c_1(h)$ and $c_3(h)$. Nevertheless, health achievement in distribution B is higher than in distribution A if one uses the scaling function $c_2(h)$.

Table 1: Distributions of health by socioeconomic statuses

Socioeconomic rank	Health status A	Health status B
1	poor	poor
2	fair	fair
3	good	good
4	good	very good
5	very good	very good
6	excellent	excellent
7	very good	excellent
8	fair	poor
9	excellent	excellent
10	poor	poor

[7] $\nu = 2$ is the most frequently used value of the parameter of aversion to socioeconomic health inequality. It is the level of aversion associated with the well known health concentration index.

Table 2: Alternative scaling functions

	$c_1(h)$	$c_2(h)$	$c_3(h)$
Poor	1	1	1
Fair	2	10	2
Good	3	11	3
Very good	4	12	4
Excellent	5	13	10

The above example illustrates that Wagstaff's health achievement indices' rankings of bivariate distributions of socioeconomic status and health may not be robust across different scales and thus may be arbitrary. This being said, it is important to mention that in some cases the ranking between two distributions may remain the same for all scaling functions $c(h)$ however the conditions for which this is true are not yet available in the literature. In the following section we develop these conditions.

4 Robust Wagstaff Orderings

The objective of this section is to present a methodology that identifies these robust rankings of Wagstaff's health achievements and socioeconomic health inequality indices. In doing so, we will consider three cases. In the first case, we will consider all types of scaling functions. In order to be consistent with the ordinal ranking of the categorical variable these scaling functions need to be monotonic and increasing. In the second case, we will consider all concave scaling functions. This case is interesting if the analyst believes that the differences between adjacent categories becomes less important when we move towards the highest category. The last case focusses on all convex scaling functions. This type of scaling function is useful when the analyst believes that the difference between adjacent categories becomes more more important when we move towards the highest category.

4.1 Health Achievement in Self-Reported Health Status

To address an issue that is akin to the one we are addressing in this paper, Allison and Foster (2004) identify robust rankings of the average of distributions of self-reported health statuses in the dimension of *pure health* inequality. They show that if a distribution of self-reported health statuses stochastically dominates another, then the average of this first distribution is higher than the average of the other distribution for all scaling functions $c(h)$. This means that if the cumulative proportion of individuals with self-reported health status below a specified threshold is lower in the first distribution for all possible thresholds, then the average of the self-reported health statuses in the first distribution is unambiguously higher than in the second. The approach proposed in this paper builds on this insight by extending it to the dimension of socioeconomic health inequality. It exploits the equivalency between Wagstaff's achievement index and the average level of health status when an equal social weight is given to all individuals regardless of their socioeconomic statuses (i.e., when the inequality aversion parameter ν, is equal to 1). It then introduces aversion to socioeconomic health inequalities by increasing the values of ν. To identify robust rankings of health achievement indices we examine the cumulative weighted proportion of the population that falls under a given health status .[8]

Formally, let $\mathscr{P}_k := \{i : h_i = k\}$ denote the set of individuals with a self-reported health status in the kth category and $\phi(k;\nu) = \sum_{i\in\mathscr{P}} \omega(r_i;\nu)$ a function that indicates the proportion of total social weight of individuals in the health category k. We define $\Phi^1(k;\nu)$ a function that plays the same role as the cumulative distribution function in the identification of first order stochastic dominance. It can be interpreted as a "socially weighted" cumulative distribution function of self-reported health statuses. It mimics the cumulative distribution that would be obtained if the social weights are considered as the probability of drawing an individual with this particular combination of socioeconomic status and self-reported health status. More specifically, $\Phi^1(k;\nu)$ indicates the

[8] Whenever $\nu > 1$, the weights of individuals with different socioeconomic statuses will be different.

total social weight for individuals with a self-reported health status lower or equal to k. It is defined as $\Phi^1(k;\nu) = \sum_{l=1}^{k} \phi(k;\nu)$. In this context, one can determine if the health achievement index of a distribution is robustly higher than the health achievement index of another distribution by comparing their socially weighted cumulative distributions.[9]

Theorem 1 *$A_1(\nu) \geq A_0(\nu)$ for all scaling functions $c(h)$ if and only if:*

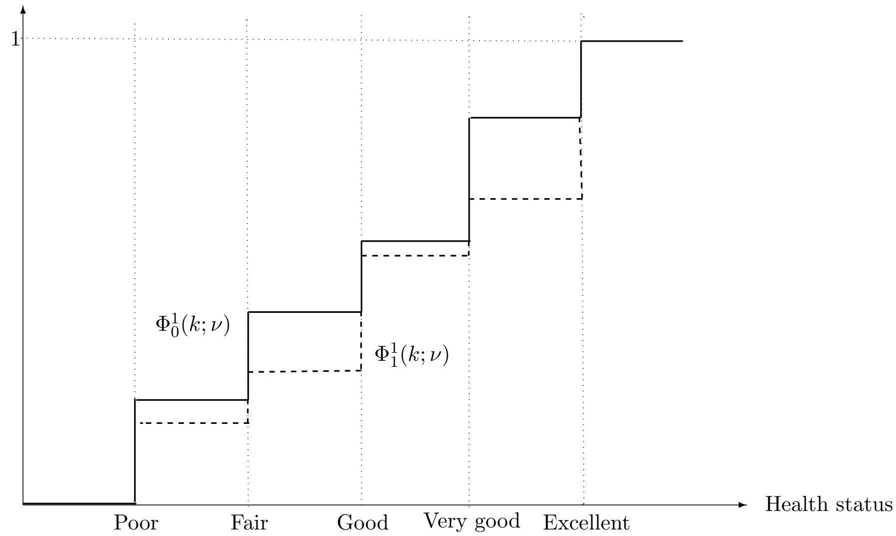
$$\Phi_0^1(k;\nu) \geq \Phi_1^1(k;\nu), \text{ for all } k \in \{1, 2, \ldots, K-1\}.$$

The condition described in Theorem 1 can be illustrated in a simple graphical test. Figure 1 illustrates the condition. If distribution 1 has a lower socially weighted cumulative distribution ($\Phi_1^1(k;\nu)$) than the other ($\Phi_0^1(k;\nu)$), then its health achievement index is unambiguously higher. Note that Theorem 1 indicates that it there is no need to test that $\Phi_0^1(K;\nu) \geq \Phi_1^1(K;\nu)$. This is due to the fact that these two values are, by definition, both equal to 1, as illustrated on the figure.

One might ask, what if at some level of socioeconomic health inequality aversion, ν, the condition of Theorem 1 does not hold? If this occurs, then two routes may be followed to circumscribe this problem. The first solution consists in increasing the level of aversion. If one increases enough the level of socioeconomic health inequality aversion, a ranking of the distribution is always obtained since there exist a complete ranking of all distribution based on the health status of the poorest individuals when $\nu \to \infty$. The second solution may exist if one is ready to impose some level of cardinality on the self-reported health status. The only level of cardinality that is required is at the level of the second derivative of the scaling function, that is the scaling function $c(h)$ is either concave or convex.

[9]For expositional ease, all proofs of the theorems are in the appendix

Figure 1: Theorem 1



If the analyst has a strong belief that differences between adjacent categories become less important as one moves towards the highest category, then it is reasonable to assume that scaling function $c(h)$ is concave. This concavity assumption is analogous to concavity of utility functions in second order stochastic dominance. However, individuals' social weights play the same role as different probabilities assigned to different events. To test for robust Wagstaff rankings of health achievement index in presence of concave scaling functions, we need to define $\Phi^{2+}(k;\nu) = \sum_{j=1}^{k} \Phi^1(j;\nu)$. One can determine if the health achievement index of a distribution is robustly higher than the health achievement index of another distribution by comparing their $\Phi^{2+}(k;\nu)$ functions. Formally,

Theorem 2 *$A_1(\nu) \geq A_0(\nu)$ for all concave scaling functions $c(h)$ if and only if:*

$$\Phi^{2+}_0(k;\nu) \geq \Phi^{2+}_1(k;\nu), \text{ for all } k \in \{1, 2, \ldots, K-1\}.$$

Theorem 2 states that if one distribution has a lower $\Phi^{2+}(k;\nu)$ than the other, then its health achievement index is unambiguously higher for all concave scaling functions. As in Theorem 1, the condition can be verified with a very simple graphical test.

Let's now turn our attention to the case of a convex scaling function $c(h)$. In this context, the analyst has a strong belief that the difference between adjacent categories becomes more important when we move towards the highest category. In order to present the test for robust rankings of a Wagstaff health achievement index in presence of convex scaling functions, we need to define $\Phi^{2-}(k;\nu) = \sum_{j=k}^{K-1} \Phi^1(j;\nu)$. One can determine if the health achievement index of a distribution is robustly higher than the health achievement index of another distribution by comparing their $\Phi^{2-}(k;\nu)$ functions. Formally,

Theorem 3 *$A_1(\nu) \geq A_0(\nu)$ for all convex scaling functions $c(h)$ if and only if:*

$$\Phi_0^{2-}(k;\nu) \geq \Phi_1^{2-}(k;\nu), \text{ for all } k \in \{1, 2, \ldots, K-1\}.$$

Theorem 3 states that if one distribution has a lower $\Phi^{2-}(k;\nu)$ than the other, then its health achievement index is unambiguously higher for all concave scaling functions. Once again, the condition remains a simple graphical test.

4.2 Inequality in self-reported health statuses

Allison and Foster (2004) argue that the mean is a non-robust reference point to assess inequality in self-reported health statuses or any other categorical variable. They argue that the standard inequality principle of aversion to mean-preserving spreads is meaningless and propose to use the median as an alternative reference point. This idea was exploited and labeled as *aversion to median preserving spread* by Abul-Naga and Yalcin (2008).

The non-robustness of the mean as a reference point can be easily understood. Consider the examples in Tables 1 and 2. The mean of distribution A is 3 under $c_1(h)$, 9.4 under $c_2(h)$ and 4 under $c_3(h)$. In the first case, the mean of the distribution corresponds to a "good" health status, in the second case, it is slightly lower than a "fair" health status and in the last case, it represents a "very good" health status. Applying different numerical scales on the same categorical information changes not only the numerical value of the mean but also the corresponding health category. For this reason, we follow Allison and Foster (2004), Abul-Naga and Yalcin (2008) and adopt the median as a reference point and use aversion to median preserving spread as a measure to define the notion of inequality in self-reported health status. A natural candidate for a statistic that captures the dispersion of the distribution of self-reported health statuses from its median is *Average Absolute Deviation about the Median*, $E|c(h) - \text{median}_c|$. To capture the socioeconomic dimension of health inequalities, one can use the insight of the previous section and therefore use Wagstaff's social weights as if they were representing a probability of observing an individual with this socioeconomic status. In this context, the index of socioeconomic inequalities in self-reported health statuses becomes:

$$I(\nu) = \sum_{i=1}^{N} \omega(r_i; \nu) \left| c(h_i) - \text{median}_c \right| . \qquad (3)$$

The inspection of equation (3) indicates that the values of this index belongs to the interval $[0, K(c(h), \nu)]$, where $K(c(h); \nu)$ is a constant specific for each scaling function and each value of the parameter of aversion to socioeconomic health inequality. To illustrate, consider scaling functions examples in Table 2. If we apply a linear scaling function $c_1(h)$, the maximum value of the index would be reached for a population of very large size (think of $N \to \infty$) when 50% of the population minus one person have a "poor" health status, one person has a "good" health status and the rest of the population has an "excellent" health status. In this case, the value of $I(2) = 2$ and this regardless of the socioeconomic ranks of the persons in each categories (this is because, for this particular scaling function, $|c(\text{excellent}) - c(\text{good})| = |c(\text{poor}) - c(\text{good})|$). If we change the scaling function and apply $c_2(h)$, the maximum value of the index is reach for a population of very large size when the 50% of the poorest population minus one person have a "poor" health status, the following person in the socioeconomic ranks has a "very good" health status and the rest of the population has an "excellent" health status. In this case, $\lim_{N\to\infty} I(2) = 11 \int_0^{0.5} \nu(1-p)^{\nu-1}dp + \int_{0.5}^{1} \nu(1-p)^{\nu-1}dp = 8.5$. Finally, if we consider the function $c_3(h)$, the maximum value of the index is reach for a population of very large size when the 50% of the poorest population have an "excellent" health status, the following person in the socioeconomic ranks has a "fair" health status and the rest of the population has a "poor" health status. In this case, $\lim_{N\to\infty} I(2) = \int_0^{0.5} \nu(1-p)^{\nu-1}dp + 8\int_{0.5}^{1} \nu(1-p)^{\nu-1}dp = 2.75$. To constrain the index values to the interval $[0,1]$, an alternative to (3) would be

$$\overline{I}(\nu) = \frac{1}{K(c(h);\nu)} \sum_{i=1}^{N} \omega(r_i;\nu)\left|c(h_i) - \text{median}_c\right|. \tag{4}$$

Given that our objective is to identify rankings of bivariate distributions of health and socioeconomic status that remain the same for all scaling functions and since the indices in (3) and (4) lead both to the same exact rankings of these distributions the theorems of this section will be applicable to both class of indices since $\overline{I}_1(\nu) > \overline{I}_0(\nu)$ whenever $I_1(\nu) > I_0(\nu)$.

For each bivariate distribution of socioeconomic statuses and self-reported health statuses, the value of $I(\nu)$ will be contingent to the choice of the scaling function. In some cases, a ranking between two distributions may remain the same for all scaling functions $c(h)$, however this is not always the case. Thus comparing two distributions according to their ranking of $I(\nu)$ may be hazardous. The objective of this subsection is to present a methodology aiming at identifying robust rankings. This methodology allows for the comparison of distributions that have similar median of health statuses. While this might appear as a limitation of the applicability of this methodology, our empirical illustration (as well as Allison and Foster's 2004) show that in practice distribution within same countries and even between countries have often very similar median category.

Let us define a functions that will play the same role as the socially weighted cumulative distribution in the previous subsection. Let $\Psi^L(k;\nu) = \sum_{j=1}^{k} \phi(j;\nu)$ and $\Psi^R(k;\nu) = \sum_{j=k}^{K} \phi(j;\nu)$, where L and R designate left and right respectively. A ranking of socioeconomic health inequality $I(\nu)$ between two distributions that is valid for all scaling functions $c(h)$ can be obtained by comparing their $\Psi^{1L}(k;\nu)$ and $\Psi^{1R}(k;\nu)$ curves. Let m represent the rank of the median category, we get:

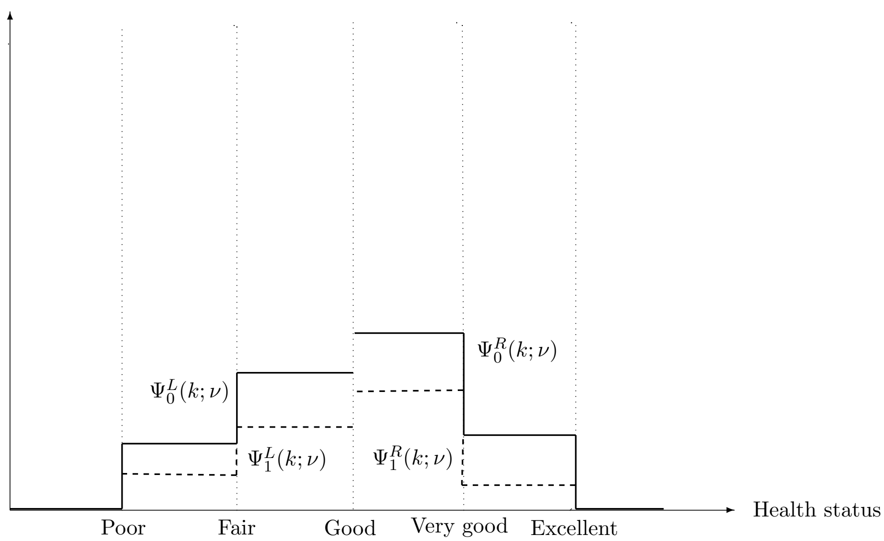
Theorem 4 *If distributions 0 and 1 have the same median category then* $I_1(\nu) \leq I_0(\nu)$ *for all* *scaling functions $c(h)$ if and only if:*

$$\Psi_0^L(k;\nu) \geq \Psi_1^L(k;\nu),\ \textit{for all } k \in \{1, 2, \ldots, m-1\}$$

and,

$$\Psi_0^R(k;\nu) \geq \Psi_1^R(k;\nu),\ \textit{for all } k \in \{m+1, m+2, \ldots, K\}$$

Figure 2: Theorem 4



The graphical form for Theorem 4 test is illustrated in Figure 2.[10] It shows that if one distribution has a lower Ψ^L function than another on the left of the median category and a lower Ψ^R function at the right of the median category, then its socioeconomic inequality in self-reported health status as measured by $I(\nu)$ is unambiguously lower.

Assume that, at some level of socioeconomic health inequality aversion, ν, the condition of Theorem 4 does not hold. As in the previous subsection, two routes may be followed to circumscribe this problem. If the second route is chosen and if the analyst has strong reasons to believe that the differences between adjacent categories become less important as one moves towards the highest

[10] In this illustrative example, we use the category that falls in the middle of the categories as the median of the 2 distributions. This choice was motivated for ease of graphical illustration. Note that the median is the median of the distribution of health statuses and need not to be the category in the middle of the health categories.

category, then the scaling function $c(h)$ needs to be concave. To present a formal test for robust rankings of the socially weighted average absolute deviation about the median in presence of concave scaling functions, we need to define $\Psi^{2+}(k;\nu)$ as

$$\Psi^{2+}(k;\nu) = \begin{cases} \sum_{j=1}^{k} \Psi^{L}(j;\nu) & \text{for } k \in \{1, 2, \dots, m-1\} \\ \Psi^{2+}(m-1;\nu) + \sum_{j=m+1}^{k} \Psi^{R}(j;\nu) & \text{for } k \in \{m+1, m+2, \dots, K\} \end{cases} \tag{5}$$

One can determine if the socially weighted average absolute deviation about the median of a distribution is robustly higher than the socially weighted average absolute deviation about the median of another distribution by comparing their $\Psi^{2+}(k;\nu)$ functions. Formally,

Theorem 5 *If distributions 0 and 1 have the same median category then* $I_1(\nu) \leq I_0(\nu)$ *for all concave scaling functions* $c(h)$ *if and only if:*

$$\Psi_0^{2+}(k;\nu) \geq \Psi_1^{2+}(k;\nu), \text{ for all } k \in \{1, 2, \dots, m-1, m+1, \dots, K\}$$

Conditions of Theorem 5 are less demanding than those of Theorem 4 as Theorem 5 exploits the information obtained by imposing concavity of the scale used. Given concavity, improvements below the median are more important than improvements above the median, then a favourable rearrangement below the median, leading to a movement towards the median, can compensate for an unfavourable rearrangement above the median, leading to a movement away from the median.

If the analyst has strong reasons to believe that the difference between adjacent categories becomes more important when we move towards the highest category, then the scaling function, $c(h)$ needs to be convex. To present a formal test for robust rankings of the socially weighted average absolute deviation about the median in presence of convex scaling functions, we need to define $\Psi^{2-}(k;\nu)$ as

$$\Psi^{2-}(k;\nu) = \begin{cases} \Psi^{2-}(m+1;\nu) + \sum_{j=k}^{m-1} \Psi^{L}(j;\nu) & \text{for } k \in \{1, 2, \dots, m-1\} \\ \sum_{j=k}^{K} \Psi^{R}(j;\nu) & \text{for } k \in \{m+1, m+2, \dots, K\} \end{cases} \tag{6}$$

One can determine if the socially weighted average absolute deviation about the median of a distribution is robustly higher than the socially weighted average absolute deviation about the median of another distribution by comparing their $\Psi^{2-}(k;\nu)$ functions. Formally,

Theorem 6 *If distributions 0 and 1 have the same median category then $I_1(\nu) \leq I_0(\nu)$ for all convex scaling functions $c(h)$ if and only if:*

$$\Psi_0^{2-}(k;\nu) \geq \Psi_1^{2-}(k;\nu),\ \textit{for all } k \in \{1,2,\ldots,m-1,m+1,\ldots,K\}$$

Conditions of Theorem 6 is less demanding than Theorem 4 as Theorem 6 exploits information obtained by imposing convexity of the scale function. Given convexity, a favourable rearrangement above the median compensates for an unfavourable rearrangement below the median. It is important to note the consequence of imposing convexity and concavity translates into opposite adjustments when favourable and unfavourable rearrangements occur below and above the median.

5 Empirical Illustration

5.1 Data

In this section, we provide an illustration that shows how the theoretical results developed earlier can be implemented empirically. We will focus on regional comparisons of self-reported health status using the National Institute of Health Survey (NHIS).[11] The NHIS monitored health outcomes of Americans since 1957. It is a cross sectional household interview survey that is representative of american households and non-institutionalized individuals. It contains data on a broad range of health topics that are collected via personal household interviews. We utilize 2012 public use files without imposing any restrictions on the sample size, therefore the total sample size at hand contains approximately $100,000$ observations representing individuals of all ages, which is around 44,000 households. We use family income to infer the socioeconomic rank of individuals. Family income is adjusted using the square root of household size. To compute socioeconomic weights we sort data by income in ascending order, then generate a variable indicating the relative position of each individual by summing all statistical weights and finally, normalizing the statistical weight by the resulting total statistical weight. Using the relative position of each individual we compute the

[11] This method can be applied for between country comparisons as well.

ethical weight as in equation (2). Given that we do not have the entire population and that we do not have equiprobable sampling weights, we computed the ethical weight as follows:

$$\tilde{\omega}(i,\nu) = (1 - p_{i-1})^{\nu} - (1 - p_i)^{\nu}, \tag{7}$$

where p_i is the relative position of an individual 'i" in the income distribution. In our empirical illustration we replace $\omega(i,\nu)$ by $\tilde{\omega}(i,\nu)$. We compute these ethical weights using 5 different values for the inequality aversion parameter ν (i.e., 1,1.5, 2, 2.5 and 3.0).[12] We then provide results of the application of Theorems 1 to 6 across US census regions using a dominance matrix.

5.2 Results

Results presented in Tables 3 to 5 provide robust Wagstaff orderings of health achievements for self-reported health status using Theorems 1 to 3 and data from US regions. We will start out by looking at the set of all monotonically increasing scale functions, we will then focus on two subsets: the subset of monotonically increasing concave scale functions and subset of monotonically increasing convex scale functions. Based on the results for the monotonically increasing scale functions (Table 3) and focusing on a setting where there is no aversion to socioeconomic inequality (i.e., compare average health status), it is clear that average health in the South region is lower than average health in the Midwest, the Northeast and the West. This result is in line with results obtained by Allison and Foster (2004) using 1994 NHIS data however, the orderings obtained are not as complete as theirs. In fact, the only additional information that we can state as far as orderings are concerned is that the Midwest is dominated by the Northeast. When we introduce socioeconomic inequality aversion, the Southern states' health achievements remain dominated by the Northeast, the Midwest and the West for all values of inequality aversion parameter up to $\nu = 2.5$, the West dominates the Midwest for $\nu > 1.5$, and finally the Midwest's health achievements are dominated by the Northeast and the West for all values of ν .

[12]The reader should keep in mind that when $\nu = 1$ there is not aversion to socioeconomic inequality.

Results for monotonically increasing concave scale functions are presented in Table 4. They show that dominance results when there is no aversion to socioeconomic health inequality are fairly similar to the ones obtained in the monotonically increasing scale function. When we introduce socioeconomic aversion to health inequality, the Southern states' health achievements remain dominated by the Northeast, the Midwest and the West for all values of ν. We also notice that the Midwest's health achievements are dominated by the Northeast and the West for all values of ν. Turning to monotonically increasing convex scale functions results presented in Table 5, we first focus on a setting where there is no aversion to socioeconomic inequality. As expected, health achievements in the South region are dominated by health achievements in the Northeast, the Midwest and the West. As discussed earlier this result is in line with previous results obtained by Allison and Foster (2004). When we introduce socioeconomic inequality aversion, the Southern states' health achievements remain dominated by the Northeast, the Midwest and the West for all values of inequality aversion parameter up to $\nu = 2.5$. We also notice that the West dominates the Midwest and that the Midwest's health achievements are dominated by the Northeast and the West for all values of ν. It should be noted at this point that since we are looking for robust orderings, it may be possible to match some Allison and Foster's results however, the orderings obtained are not as complete as theirs. Indeed obtaining robust orderings while considering the socioeconomic dimension comes at the cost of incomplete ordering. More specifically, Allison and Foster (2004) focus on robust rankings in the context of pure health inequality (i.e., for $\nu = 1$), in this paper we focus on robust orderings in the dimension of socioeconomic health inequality (i.e.,for $\nu \geq 1$).

Socioeconomic health inequality comparisons covering all US regions and a wide spectrum of values for aversion to inequality parameter are reported in Tables 6 to 8. At first glance, results do not show a clear patterns resembling the ones obtained for health achievements. This kind of discrepancy between results obtained when computing achievement indices and inequality indices is due to the fact that inequality comparisons overlook the average level of health. This finding is in line with what has been found in the income inequality literature. In fact Shorrocks (1983) argues that it is easier to obtain general Lorenz rankings for welfare than Lorenz ranking for inequality. In the same spirit, Wagstaff (2002) show that two distributions may have the same relative distribution (thus the same value of the health concentration index) but one may have a higher average level of health that leads to a higher value of the achievement index. As a result, one will obtain more ranking when using an achievement index than when using an inequality index.

As earlier, we will start discussing results for monotonically increasing scale functions then focus on two subsets: monotonically increasing concave scale functions and monotonically increasing convex scale functions. Based on the results in Table 6 and in a setting where there is no aversion to socioeconomic inequality we notice that there are no clear dominance patterns. As in the case of health achievements, this result is in line with results obtained by Allison and Foster (2004) however, dominance results are different from Allison and Foster (2004) as the West seems to be dominated by the Northeast. When we introduce socioeconomic inequality aversion, the West is dominated by the Northeast for all values of ν up to $\nu = 2$. Table 7 present results for a monotonically increasing concave scale function. A first inspection of the obtained results reveals that health achievements in the South are dominated by health achievements in the Northeast and the Midwest, and that the Midwest was dominated by the Northeast up to $\nu = 2.5$. Finally looking of the monotonically increasing and the convex scale case (i.e., Table 8) in a setting where there is no aversion to socioeconomic inequality, the results reveal no clear dominance patterns. When we introduce socioeconomic inequality aversion, we notice that the West is dominated by the Northeast for all value of ν.

In this section we illustrated how, using Theorems 1 to 6, a researcher can make robust comparisons of health distributions (achievements and socioeconomic inequalities) without imposing a specific numerical scale. Generally speaking, results from the empirical illustration seem to indicate that robust orderings are difficult to identify without imposing a specific numerical scale. This suggests that robust orderings might come at the cost of incomplete rankings and unclear dominance patterns. While the researcher might be able to draw fewer conclusions when using this approach, all the orderings obtained will be robustly identified.

6 Conclusion

In this paper, we address an important measurement problem that arises when using categorical data to conduct socioeconomic health inequality analysis: *the arbitrariness of the concentration index*. In doing so, we build on Allison and Foster (2004) and extend their analysis to account for the socioeconomic dimension of health inequality. We construct a socially weighted cumulative distribution of self-reported health status that mimics the cumulative distribution that would be obtained if social weights were considered as a probability of drawing an individual with a particular socioeconomic health status and self-reported health status. Thus by using the cumulative weighted proportion of population that falls under a given health status we derive conditions that allows us to produce robust Wagstaff orderings of health achievement and socioeconomic health inequality indices. This approach allows us to use categorical variable as they are without imposing any explicit numerical scale. To obtain more complete orderings researcher may be willing to make some assumption on the general form of the scale function. We visit this possibility and provide the conditions for which a concave and convex scale function provide robust Wagstaff orderings. While the proposed method presents the advantage of allowing the researcher to use categorical variables and obtain robust rankings that account for the depth of the health problems as well as the socioeconomic dimension on health inequality, our method (as Allison and Foster's) is only applicable when medians of the distributions compared are identical. While this is often not an issue, it remains important to insure that this condition is verified. Another, issue that may arise when applying this method is that it often provides incomplete rankings. This does not comes as a surprise as the robustness of the results is often decreasing in the number of the assumptions we are ready to make. One natural extension of this paper would be to generalize the tests to account for all possible weights functions ω and not just for one specific Wagstaff function with a specific ν. This may be done building on the multidimensional inequality dominance literature.

A Appendix

A.1 Proof of Theorem 1

Proof. Decomposing equation (1), we obtain:

$$A(\nu) = c(h_K) - \sum_{k=1}^{K-1} [c(h_{k+1}) - c(h_k)] \sum_{j=1}^{k} \sum_{i \in p_j} \omega(r_i; \nu). \tag{8}$$

Replacing $\Phi^1(k;\nu) = \sum_{j=1}^{k} \sum_{i \in p_j} \omega(r_i;\nu)$ and $\Delta^1 c(h_k) = c(h_{k+1}) - c(h_k)$ in (8), we get

$$A(\nu) = c(h_K) - \sum_{k=1}^{K-1} \Delta^1 c(h_k) \Phi^1(k;\nu). \tag{9}$$

Using equation (9), we get:

$$A_1(\nu) - A_0(\nu) = \sum_{k=1}^{K-1} \left[\Phi_0^1(k;\nu) - \Phi_1^1(k;\nu)\right] \Delta^1 c(h_k). \tag{10}$$

In order to represent the ordinal self-reported health status variable, we must assume that the scaling function is at least weakly monotonically increasing, i.e. $\Delta^1 c(h_k) \geq 0$. This implies that if $\Phi_0^1(k;\nu) \geq \Phi_1^1(k;\nu)$ for all $k \in \{1, 2, \ldots, K-1\}$, then the expression in equation (10) is non negative, i.e. $A_1(\nu) \geq A_0(\nu)$. This proves sufficiency of the condition.

In order to establish necessity, let us consider a particular scaling function that has the following properties:

1. $c(h_1) = c(h_2) = \cdots = c(h_{k^*})$

2. $\Delta^1 c(h_{k^*}) > 0$

3. $c(h_{k^*+1}) = c(h_{k^*+2}) = \cdots = c(h_{K-1})$

Imagine now that $\Phi_0^1(k;\nu) \geq \Phi_1^1(k;\nu)$ for all k excepts for k^* for which we have $\Phi_0^1(k^*;\nu) < \Phi_1^1(k^*;\nu)$. For any scaling function having the above mentioned properties, $A_0(\nu) > A_1(\nu)$. Hence it cannot be that $\Phi_0^1(k;\nu) < \Phi_1^1(k;\nu)$ for any $k \in \{1, 2, \ldots, K-1\}$. This proves the necessity of the condition.

■

A.2 Proof of Theorem 2

Proof. One decomposition of equation (9) yields:

$$A(\nu) = c(h_K) - \Delta^1 c(h_{K-1}) \sum_{k=1}^{K-1} \Phi^1(k;\nu) + \sum_{k=1}^{K-2} \left[\Delta^1 c(h_{k+1}) - \Delta^1 c(h_k)\right] \sum_{j=1}^{k} \Phi^1(k;\nu). \tag{11}$$

Replacing $\Phi^{2+}(k;\nu) = \sum_{j=1}^{k} \Phi^1(k;\nu)$ and $\Delta^2 c(h_k) = \Delta^1 c(h_{k+1}) - \Delta^1 c(h_k)$ in equation (11), yields to:

$$A(\nu) = c(h_K) - \Phi^{2+}(K-1;\nu)\Delta^1 c(h_{K-1}) + \sum_{k=1}^{K-2} \Phi^{2+}(k;\nu)\Delta^2 c(h_k). \tag{12}$$

Using equation (12), we get:

$$A_1(\nu) - A_0(\nu) = \left[\Phi_0^{2+}(K-1;\nu) - \Phi_1^{2+}(K-1;\nu)\right] \Delta^1 c(h_{K-1}) + \sum_{k=1}^{K-2} \left[\Phi_1^{2+}(k;\nu) - \Phi_0^{2+}(k;\nu)\right] \Delta^2 c(h_k) \tag{13}$$

If we consider only concave scaling functions to represent the self-reported health status variable, then $\Delta^2 c(h_k) \leq 0$ in addition to $\Delta^1 c(h_k) \geq 0$. This implies that if $\Phi_0^{2+}(k;\nu) \geq \Phi_1^{2+}(k;\nu)$ for all $k \in \{1, 2, \ldots, K-1\}$, then the expression in equation (13) is non negative, i.e. $A_1(\nu) \geq A_0(\nu)$. This proves sufficiency of the condition.

In order to establish necessity, we have to proceed in two steps. First consider a linear scaling function. In this case, $\Delta^2 c(h_k) = 0$ for all k. Imagine now that $\Phi_0^{2+}(k;\nu) \geq \Phi_1^{2+}(k;\nu)$ for all $k \in \{1, 2, \ldots, K-2\}$ and that for $K-1$, we have $\Phi_0^{2+}(K-1;\nu) < \Phi_1^{2+}(K-1;\nu)$. For these linear scaling functions, $A_0(\nu) > A_1(\nu)$. Hence it cannot be that $\Phi_0^{2+}(K-1;\nu) < \Phi_1^{2+}(K-1;\nu)$. Let us now consider a particular scaling function that has the following properties:

1. $\Delta^1 c(h_1) = \Delta^1 c(h_2) = \cdots = \Delta^1 c(h_{k^*}) > 0$

2. $\Delta^1 c(h_{k^*+1}) = \Delta^1 c(h_{k^*+2}) = \cdots = \Delta^1 c(h_{K-1}) = 0$

Imagine now that $\Phi_0^{2+}(k;\nu) \geq \Phi_1^{2+}(k;\nu)$ for all k excepts for k^* for which we have $\Phi_0^{2+}(k^*;\nu) < \Phi_1^{2+}(k^*;\nu)$. For any scaling function having the above mentioned properties, $A_0(\nu) > A_1(\nu)$. Hence it cannot be that $\Phi_0^1(k;\nu) < \Phi_1^1(k;\nu)$ for any $k \in \{1, 2, \ldots, K-1\}$. This proves the necessity of the condition. ■

A.3 Proof of Theorem 3

Proof. Another decomposition of equation (9) yields:

$$A(\nu) = c(h_K) - \Delta^1 c(h_1) \sum_{k=1}^{K-1} \Phi^1(k;\nu) - \sum_{k=2}^{K-1} \left[\Delta^1 c(h_{k+1}) - \Delta^1 c(h_k)\right] \sum_{j=k}^{K-1} \Phi^1(k;\nu). \qquad (14)$$

Replacing $\Phi^{2-}(k;\nu) = \sum_{j=k}^{K-1} \Phi^1(k;\nu)$ and $\Delta^2 c(h_k) = \Delta^1 c(h_{k+1}) - \Delta^1 c(h_k)$ in equation (14), yields to:

$$A(\nu) = c(h_K) - \Phi^{2-}(1;\nu)\Delta^1 c(h_1) - \sum_{k=1}^{K-2} \Phi^{2+}(k;\nu)\Delta^2 c(h_k). \qquad (15)$$

Using equation (15), we get:

$$A_1(\nu) - A_0(\nu) = \left[\Phi_0^{2-}(1;\nu) - \Phi_1^{2-}(1;\nu)\right] \Delta^1 c(h_1) - \sum_{k=2}^{K-1} \left[\Phi_0^{2-}(k;\nu) - \Phi_1^{2-}(k;\nu)\right] \Delta^2 c(h_k) \qquad (16)$$

If we consider only convex scaling functions to represent the self-reported health status variable, then $\Delta^2 c(h_k) \geq 0$ in addition to $\Delta^1 c(h_k) \geq 0$. This implies that if $\Phi_0^{2-}(k;\nu) \geq \Phi_1^{2-}(k;\nu)$ for all $k \in \{1, 2, \ldots, K-1\}$, then the expression in equation (16) is non negative, i.e. $A_1(\nu) \geq A_0(\nu)$. This proves sufficiency of the condition.

In order to establish necessity, we have to proceed in two steps. First consider a linear scaling function. In this case, $\Delta^2 c(h_k) = 0$ for all k. Imagine now that $\Phi_0^{2-}(k;\nu) \geq \Phi_1^{2-}(k;\nu)$ for all $k \in \{2, \ldots, K-1\}$ and that for $k = 1$, we have $\Phi_0^{2-}(1;\nu) < \Phi_1^{2-}(1;\nu)$. For these linear scaling functions, $A_0(\nu) > A_1(\nu)$. Hence it cannot be that $\Phi_0^{2-}(1;\nu) < \Phi_1^{2-}(1;\nu)$. Let us now consider a particular scaling function that has the following properties:

1. $\Delta^1 c(h_1) = \Delta^1 c(h_2) = \cdots = \Delta^1 c(h_{k^*}) = 0$

2. $\Delta^1 c(h_{k^*+1}) = \Delta^1 c(h_{k^*+2}) = \cdots = \Delta^1 c(h_{K-1}) > 0$

Imagine now that $\Phi_0^{2-}(k;\nu) \geq \Phi_1^{2-}(k;\nu)$ for all k excepts for k^* for which we have $\Phi_0^{2-}(k^*;\nu) < \Phi_1^{2-}(k^*;\nu)$. For any scaling function having the above mentioned properties, $A_0(\nu) > A_1(\nu)$. Hence it cannot be that $\Phi_0^1(k;\nu) < \Phi_1^1(k;\nu)$ for any $k \in \{1, 2, \ldots, K-1\}$. This proves the necessity of the condition. ■

A.4 Proof of Theorem 4

Proof. Equation (3) can be rewritten as

$$I(\nu) = \sum_{k=1}^{m-1} \sum_{i \in p_k} \omega(r_i;\nu)(c(h_m) - c(h_k)) + \sum_{k=m+1}^{K} \sum_{i \in p_k} \omega(r_i;\nu)(c(h_k) - c(h_m)). \tag{17}$$

Decomposing the two terms on the right hand side of equation (17) yields:

$$I(\nu) = \sum_{k=1}^{m-1} (c(h_{k+1}) - c(h_k)) \sum_{j=1}^{k} \sum_{i \in p_k} \omega(r_i;\nu) + \sum_{k=m+1}^{K} (c(h_k) - c(h_{k-1})) \sum_{j=k}^{K} \sum_{i \in p_k} \omega(r_i;\nu). \tag{18}$$

Replacing $\Psi^L(k;\nu) = \sum_{j=1}^{k} \sum_{i \in p_k} \omega(r_i;\nu)$, $\Psi^R(k;\nu) = \sum_{j=k}^{K} \sum_{i \in p_k} \omega(r_i;\nu)$ and $\Delta^1 c(h_k) = c(h_{k+1}) - c(h_k)$ in equation (18), yields:

$$I(\nu) = \sum_{k=1}^{m-1} \Psi^L(k;\nu) \Delta^1 c(h_k) + \sum_{k=m+1}^{K} \Psi^R(k;\nu) \Delta^1 c(h_{k-1}). \tag{19}$$

Using equation (19), we get

$$I_1(\nu) - I_0(\nu) = \sum_{k=1}^{m-1} \left[\Psi_1^L(k;\nu) - \Psi_0^L(k;\nu)\right] \Delta^1 c(h_k) + \sum_{k=m+1}^{K} \left[\Psi_1^R(k;\nu) - \Psi_0^R(k;\nu)\right] \Delta^1 c(h_{k-1}). \tag{20}$$

Since $\Delta^1 c(h_k) \geq 0$. This implies that if $\Psi_1^L(k;\nu) \geq \Psi_0^L(k;\nu)$ for all $k \in \{1, 2, \ldots, m-1\}$ and $\Psi_1^R(k;\nu) \geq \Psi_0^R(k;\nu)$ for all $k \in \{m+1, m+2, \ldots, K\}$, then the expression in equation (20) is non positive, i.e. $I_0(\nu) \geq I_1(\nu)$. This proves sufficiency of the condition.

In order to establish necessity, let us consider the same particular scaling function that we used in the proof of Theorem 1. Imagine now that $\Psi_0^\lambda(k;\nu) \geq \Psi_1^\lambda(k;\nu)$, $\lambda = L$ or R for all k excepts for k^* for which we have $\Psi_0^\lambda(k^*;\nu) < \Psi_1^\lambda(k^*;\nu)$. For any scaling function having the properties described in the proof of Theorem 1, $I_0(\nu) < I_1(\nu)$. Hence it cannot be that $\Psi_0^\lambda(k^*;\nu) < \Psi_1^\lambda(k^*;\nu)$ for any $k \in \{1, \ldots, m-1, m+1, \ldots, K\}$. This proves the necessity of the condition. ■

A.5 Proof of Theorem 5

Proof. One decomposition of equation (19) yields:

$$\begin{aligned} I(\nu) &= \Delta^1 c(h_{K-1}) \left[\sum_{k=1}^{m-1} \Psi^L(k;\nu) + \sum_{k=m+1}^{K} \Psi^R(k;\nu) \right] \\ &+ \sum_{k=m+1}^{K} \left[\Delta^1 c(h_k) - \Delta^1 c(h_{k-1}) \right] \left[\sum_{j=1}^{m-1} \Psi^L(j;\nu) + \sum_{j=m+1}^{k} \Psi^R(j;\nu) \right] \\ &+ \sum_{k=1}^{m-1} \left[\Delta^1 c(h_{k+1}) - \Delta^1 c(h_k) \right] \sum_{j=1}^{k} \Psi^L(j;\nu) \end{aligned} \tag{21}$$

Replacing $\Delta^2 c(h_k) = \Delta^1 c(h_{k+1}) - \Delta^1 c(h_k)$ and replacing by $\Psi^{2+}(k;\nu)$ as described in equation (5) the values in equation (21), yields to:

$$I(\nu) = \Delta^1 c(h_{K-1}) \Psi^{2+}(K;\nu) - \sum_{k=m+1}^{K} \Psi^{2+}(k;\nu) \Delta^2 c(h_{k-1}) - \sum_{k=1}^{m-1} \Psi^{2+}(k;\nu) \Delta^2 c(h_k) \tag{22}$$

Using equation (22), we get

$$\begin{aligned} I_1(\nu) - I_0(\nu) &= \Delta^1 c(h_{K-1}) \left[\Psi_1^{2+}(K;\nu) - \Psi_0^{2+}(K;\nu) \right] \\ &+ \sum_{k=m+1}^{K} \left[\Psi_0^{2+}(k;\nu) - \Psi_1^{2+}(k;\nu) \right] \Delta^2 c(h_{k-1}) \\ &+ \sum_{k=1}^{m-1} \left[\Psi_0^{2+}(k;\nu) - \Psi_1^{2+}(k;\nu) \right] \Delta^2 c(h_k) \end{aligned} \tag{23}$$

If we consider only concave scaling functions to represent the self-reported health status variable, then $\Delta^2 c(h_k) \leq 0$ in addition to $\Delta^1 c(h_k) \geq 0$. This implies that if $\Psi_0^{2+}(k;\nu) \geq \Phi_1^{2+}(k;\nu)$ for all $k \in \{1, 2, \ldots, m-1, m+1, \ldots, K\}$, then the expression in equation (23) is non positive, i.e. $I_1(\nu) \leq I_0(\nu)$. This proves sufficiency of the condition.

In order to establish necessity, we have to proceed in two steps. First consider a linear scaling function. In this case, $\Delta^2 c(h_k) = 0$ for all k. Imagine now that $\Psi_0^{2+}(k;\nu) \geq \Psi_1^{2+}(k;\nu)$ for all $k \in \{1, 2, \ldots, m-1, m+1, \ldots, K-1\}$ and that for K, we have $\Psi_0^{2+}(K;\nu) < \Psi_1^{2+}(K;\nu)$. For these linear scaling functions, $I_1(\nu) > I_0(\nu)$. Hence it cannot be that $\Psi_0^{2+}(K;\nu) < \Psi_1^{2+}(K;\nu)$. Let us now consider a particular scaling function that has the following properties:

1. $\Delta^1 c(h_1) = \Delta^1 c(h_2) = \cdots = \Delta^1 c(h_{k^*}) > 0$

2. $\Delta^1 c(h_{k^*+1}) = \Delta^1 c(h_{k^*+2}) = \cdots = \Delta^1 c(h_{K-1}) = 0$

Imagine now that $\Psi_0^{2+}(k;\nu) \geq \Psi_1^{2+}(k;\nu)$ for all k excepts for k^* for which we have $\Psi_0^{2+}(k^*;\nu) < \Psi_1^{2+}(k^*;\nu)$. For any scaling function having the above mentioned properties, $I_1(\nu) > I_0(\nu)$. Hence it cannot be that $\Psi_0^1(k;\nu) < \Psi_1^1(k;\nu)$ for any $k \in \{1, 2, \ldots, m-1, m+1, \ldots, K\}$. This proves the necessity of the condition. ■

A.6 Proof of Theorem 6

Proof. One decomposition of equation (19) yields:

$$\begin{aligned} I(\nu) &= \Delta^1 c(h_1)\left[\sum_{k=1}^{m-1}\Psi^L(k;\nu) + \sum_{k=m+1}^{K}\Psi^R(k;\nu)\right] \\ &+ \sum_{k=2}^{m-1}\left[\Delta^1 c(h_k) - \Delta^1 c(h_{k-1})\right]\left[\sum_{j=k}^{m-1}\Psi^L(j;\nu) + \sum_{j=m+1}^{K}\Psi^R(j;\nu)\right] \\ &+ \sum_{k=m+1}^{K}\left[\Delta^1 c(h_{k-1}) - \Delta^1 c_(h_{k-2})\right]\sum_{j=k}^{K}\Psi^R(j;\nu) \end{aligned} \tag{24}$$

Replacing $\Delta^2 c(h_k) = \Delta^1 c(h_{k+1}) - \Delta^1 c(h_k)$ and replacing by $\Psi^{2-}(k;\nu)$ as described in equation (6) the values in equation (24), yields to:

$$I(\nu) = \Delta^1 c(h_1)\Psi^{2-}(1;\nu) + \sum_{k=2}^{m-1}\Psi^{2-}(k;\nu)\Delta^2 c(h_{k-1}) + \sum_{k=m+1}^{K}\Psi^{2-}(k;\nu)\Delta^2 c(h_{k-2}) \tag{25}$$

Using equation (25), we get

$$\begin{aligned} I_1(\nu) - I_0(\nu) &= \Delta^1 c(h_1)\left[\Psi_1^{2-}(1;\nu) - \Psi_0^{2-}(1;\nu)\right] \\ &+ \sum_{k=2}^{m-1}\left[\Psi_1^{2-}(k;\nu) - \Psi_0^{2-}(k;\nu)\right]\Delta^2 c(h_{k-1}) \\ &+ \sum_{k=m+1}^{K}\left[\Psi_1^{2-}(k;\nu) - \Psi_0^{2-}(k;\nu)\right]\Delta^2 c(h_{k-2}) \end{aligned} \tag{26}$$

If we consider only convex scaling functions to represent the self-reported health status variable, then $\Delta^2 c(h_k) \geq 0$ in addition to $\Delta^1 c(h_k) \geq 0$. This implies that if $\Psi_0^{2-}(k;\nu) \geq \Psi_1^{2-}(k;\nu)$ for all

$k \in \{1, 2, \ldots, m-1, m+1, \ldots, K-1\}$, then the expression in equation (26) is non positive, i.e. $I_1(\nu) \leq I_0(\nu)$. This proves sufficiency of the condition.

In order to establish necessity, we have to proceed in two steps. First consider a linear scaling function. In this case, $\Delta^2 c(h_k) = 0$ for all k. Imagine now that $\Psi_0^{2-}(k;\nu) \geq \Psi_1^{2-}(k;\nu)$ for all $k \in \{2, \ldots, m-1, m+1, \ldots, K\}$ and that for $k = 1$, we have $\Psi_0^{2-}(1;\nu) < \Psi_1^{2-}(1;\nu)$. For these linear scaling functions, $I_1(\nu) > I_0(\nu)$. Hence it cannot be that $\Psi_0^{2-}(1;\nu) < \Psi_1^{2-}(1;\nu)$. Let us now consider a particular scaling function that has the following properties:

1. $\Delta^1 c(h_1) = \Delta^1 c(h_2) = \cdots = \Delta^1 c(h_{k^*}) = 0$

2. $\Delta^1 c(h_{k^*+1}) = \Delta^1 c(h_{k^*+2}) = \cdots = \Delta^1 c(h_{K-1}) > 0$

Imagine now that $\Psi_0^{2-}(k;\nu) \geq \Psi_1^{2-}(k;\nu)$ for all k excepts for k^* for which we have $\Psi_0^{2-}(k^*;\nu) < \Psi_1^{2-}(k^*;\nu)$. For any scaling function having the above mentioned properties, $I_1(\nu) > I_0(\nu)$. Hence it cannot be that $\Psi_0^{1}(k;\nu) < \Psi_1^{1}(k;\nu)$ for any $k \in \{1, 2, \ldots, m-1, m+1, \ldots, K-1\}$. This proves the necessity of the condition. ■

References

[1] Allison, R.A. and J.E. Foster (2004), Measuring health inequality using qualitative data, *Journal of Health Economics*, 23, 505-524.

[2] Abul Naga, R.H. and T. Yalcin (2008), Inequality measurement for ordered response health data, *Journal of Health Economics*, 27, 1614-1625.

[3] Clarke, P.M., U.G. Gerdtham, M. Johannesson, K. Bingefors and L. Smith (2002) On the measurement of relative and absolute income-related health inequality. *Social Science and Medicine*, 55, 19231728.

[4] Costa Font, J. and F. Cowell (2013), Measuring health inequality with categorical data: some regional patterns, *Research on Economic Inequality*, 21, 53-76.

[5] Cowell, F. and E. Flachaire (2012), Inequality with ordinal data, Public Economics Programme Discussion Paper No. 16, London School of Economics.

[6] Erreygers, G. (2006), Beyond the health Concentration Index: an Atkinson alternative for the measurement of the socioeconomic inequality of health, in *Paper Presented at the Conference Advancing Health Equity*, Helsinki, WIDER-UN.

[7] Erreygers, G. (2009a), Correcting the concentration index, *Journal of Health Economics*, 28, 504-515.

[8] Erreygers, G. (2009b), Correcting the concentration index: A reply to Wagstaff, *Journal of Health Economics*, 28, 521-524.

[9] Erreygers, G., P. Clarke and T. Van Ourti (2012), “Mirror, mirror, on the wall, who is this land is the fairest of all” - Distributional sensitivity in the measurement of socioeconomic inequality in health, *Journal of Health Economics*, 31, 257-270.

[10] Erreygers, G. and T. Van Ourti (2011), Measuring socioeconomic health inequality in health, health care and health care financing by means of rank-dependent indices: A recipe for good practice, *Journal of Health Economics*, 30, 685-694.

[11] Lambert, P. and B. Zheng (2011), On the consistent measurement of attainment and shortfall inequality, *Journal of Health Economics*, 30, 214-219.

[12] Lazar, A. and J. Silber (2013), On the cardinal measurement of health inequality when only ordinal information is available on individual health status, *Health Economics*, 22, 106-113.

[13] Makdissi, P., D. Sylla and M. Yazbeck (2013), Decomposing health achievement and socioeconomic health inequalities in presence of multiple categorical information, *Economic Modelling*, 35, 964-968.

[14] Makdissi, P. and M. Yazbeck (2014), Measuring socioeconomic health inequalities in presence of multiplecategorical information, *Journal of Health Economics*, 34, 84-95.

[15] Wagstaff, A. (2002), Inequality aversion, health inequalities and health achievement, *Journal of Health Economics*, 21, 627-641.

[16] Wagstaff, A. (2005), The bounds of the concentration index when the variable of inter-est is binary, with an application to immunization inequality, *Health Economics*,14, 429-432.

[17] Wagstaff, A. (2009), Correcting the concentration index: A comment, *Journal of Health Economics*, 28, 516-520.

[18] Yitzhaki, S. (1983), On an extension of the Gini index, *International Economic Review*, 24, 617-628.

[19] Zheng, B. (2008), Measuring inequality with ordinal data, *Research on Economic Inequality*, 16, 177-188.

[20] Zheng, B. (2011), A new approach to measure socioeconomic inequality in health, *Journal of Economic Inequality*, 9, 555-577.

Table 3: Theorem 1

	Northeast	Midwest	South	West
		$\nu = 1$		
Northeast		D	D	ND
Midwest			D	ND
South				
West			D	
	Northeast	Midwest	South	West
		$\nu = 1.5$		
Northeast		D	D	ND
Midwest			D	ND
South				
West			D	
	Northeast	Midwest	South	West
		$\nu = 2$		
Northeast		D	D	ND
Midwest			D	
South				
West		D	D	
	Northeast	Midwest	South	West
		$\nu = 2.5$		
Northeast		D	D	ND
Midwest			D	
South				
West		D	D	
	Northeast	Midwest	South	West
		$\nu = 3$		
Northeast		D	D	
Midwest			ND	
South				
West	D	D	D	

Table 4: Theorem 2

	Northeast	Midwest	South	West
		$\nu = 1$		
Northeast		D	D	D
Midwest			D	ND
South				
West			D	
	Northeast	Midwest	South	West
		$\nu = 1.5$		
Northeast		D	D	D
Midwest			D	
South				
West		D	D	
	Northeast	Midwest	South	West
		$\nu = 2$		
Northeast		D	D	ND
Midwest			D	
South				
West		D	D	
	Northeast	Midwest	South	West
		$\nu = 2.5$		
Northeast		D	D	ND
Midwest			D	
South				
West		D	D	
	Northeast	Midwest	South	West
		$\nu = 3$		
Northeast		D	D	
Midwest			D	
South				
West	D	D	D	

Table 5: Theorem 3

	Northeast	Midwest	South	West
		$\nu = 1$		
Northeast		D	D	ND
Midwest			D	
South				
West		D	D	
	Northeast	Midwest	South	West
		$\nu = 1.5$		
Northeast		D	D	ND
Midwest			D	
South				
West		D	D	
	Northeast	Midwest	South	West
		$\nu = 2$		
Northeast		D	D	
Midwest			D	
South				
West	D	D	D	
	Northeast	Midwest	South	West
		$\nu = 2.5$		
Northeast		D	D	
Midwest			D	
South				
West	D	D	D	
	Northeast	Midwest	South	West
		$\nu = 3$		
Northeast		D	D	
Midwest			ND	
South				
West	D	D	D	

Table 6: Theorem 4

	Northeast	Midwest	South	West
		$\nu = 1$		
Northeast		ND	ND	D
Midwest			ND	ND
South				ND
West				
	Northeast	Midwest	South	West
		$\nu = 1.5$		
Northeast		ND	ND	D
Midwest			ND	ND
South				ND
West				
	Northeast	Midwest	South	West
		$\nu = 2$		
Northeast		ND	ND	D
Midwest			ND	ND
South				ND
West				
	Northeast	Midwest	South	West
		$\nu = 2.5$		
Northeast		ND	ND	ND
Midwest			ND	ND
South				ND
West				
	Northeast	Midwest	South	West
		$\nu = 3$		
Northeast		ND	ND	ND
Midwest			D	ND
South				ND
West				

Table 7: Theorem 5

	Northeast	Midwest	South	West
		$\nu = 1$		
Northeast		D	D	D
Midwest			D	ND
South				D
West				
	Northeast	Midwest	South	West
		$\nu = 1.5$		
Northeast		D	D	D
Midwest			D	ND
South				D
West				
	Northeast	Midwest	South	West
		$\nu = 2$		
Northeast		D	D	D
Midwest			D	
South				D
West		D		
	Northeast	Midwest	South	West
		$\nu = 2.5$		
Northeast		D	D	ND
Midwest			D	
South				D
West		D		
	Northeast	Midwest	South	West
		$\nu = 3$		
Northeast		ND	ND	ND
Midwest			D	
South				D
West		D		

Table 8: Theorem 6

	Northeast	Midwest	South	West
		$\nu = 1$		
Northeast		ND	ND	D
Midwest			ND	D
South				ND
West				
	Northeast	Midwest	South	West
		$\nu = 1.5$		
Northeast		ND	ND	D
Midwest			ND	D
South				ND
West				
	Northeast	Midwest	South	West
		$\nu = 2$		
Northeast		ND	ND	D
Midwest			ND	ND
South				ND
West				
	Northeast	Midwest	South	West
		$\nu = 2.5$		
Northeast		ND	ND	D
Midwest			ND	ND
South				ND
West				
	Northeast	Midwest	South	West
		$\nu = 3$		
Northeast		ND	ND	D
Midwest			D	ND
South				ND
West				