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Industry-level Econometric Estimates of Energy-capital-labour Substitution with a Nested CES Production Function

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Abstract

Despite substantial interest in the role of energy in the economy, the degree of substitutability between energy and other production inputs, and the way energy should be included in the production function remain unresolved issues. Our study provides industry-level parameter estimates of two-level constant elasticity of substitution (CES) functions that include capital, labour and energy inputs and allow for technological change for Canada. In contrast to many existing studies, we do not impose prior restrictions on the order of input nesting, and we report the estimates for three possible specifications. We find that a nested production structure which first combines labour and energy into a composite good that is then combined with capital, fits the Canadian data best, in terms of respecting the restrictions imposed by cost minimization. We also find rather low elasticities of substitution between capital and labour, and limited evidence of exogenous technological change.

Key words: *energy; elasticity of substitution; CES function; industry estimates*
JEL Classification: E23, Q41, Q43, Q55

Résumé

Malgré l'intérêt considérable par rapport au rôle de l'énergie dans l'économie, le degré de substituabilité entre l'énergie et autres facteurs de production, et la façon dont l'énergie devrait être introduite dans la fonction de production restent des questions non résolues. Notre étude fournit, pour le Canada, des estimations sectorielles des élasticités de substitution de fonctions de production de type CES emboîtées à deux niveaux, qui combinent le capital, le travail et l'énergie, et qui prennent en compte le changement technologique. Contrairement à de nombreuses études existantes, nous n'imposons pas de restrictions a priori sur l'ordre de l'emboîtement dans les fonctions CES, et nous présentons les estimations pour les trois spécifications possibles. Nous avons trouvé qu'une structure de production imbriquée qui allie le travail et l'énergie dans un facteur composite, qui est ensuite combiné avec le capital, correspond mieux aux données canadiennes, en ce qui a trait au respect des restrictions imposées par la minimisation des coûts. Nous avons également trouvé des élasticités de substitution relativement faibles entre le capital et le travail, et une très faible évidence pour un changement technologique exogène.

Mots clés: *énergie ; élasticité de substitution ; fonction CES ; estimations sectorielles*
Classification JEL: E23, Q41, Q43, Q55

1. Introduction

Understanding the effects of changing energy prices and the impacts of variations in the use and production of energy is important for addressing many economic and policy questions. For example, the use of energy is directly related to greenhouse gas emissions, and is thus central to the design of environmental policy. The scarcity of traditional energy sources, such as coal, oil and gas, raises concerns about sustainability and affects long-run production plans. In the short run, spikes in energy prices increase production costs and reduce aggregate demand, which in turn could lead to economic recessions.[1] The ability of an economy to respond to energy-related shocks largely depends on the extent to which energy can be substituted for other inputs of production. The main objective of this study is to provide industry-level estimates of the elasticities of substitution between energy, capital and labour.

Despite a large existing body of empirical and theoretical literature on the role of energy in the economy, there is no consensus on how the energy input should enter the production function. A constant elasticity of substitution (CES) production function is the most popular choice among theoretical modellers. In this study, we focus on aggregating capital, labour, and energy into a two-level CES function. The two-level CES specification, originally proposed by Sato (1967), has several advantages. This function has intuitive economic meaning. It is simple, and yet sufficiently flexible for capturing the substitution possibilities among input factors (Beckman and Hertel, 2010). In addition, it is well suited for formalizing a correspondence between these substitution possibilities and growth (de La Grandville, 1989; Klump and de La Grandville, 2000; de La Grandville, 2009).

Capital, labour and energy can be nested into a two-level CES function in three different ways: (KL)E, (KE)L and K(LE). The variables in the parentheses indicate the input aggregation at the lowest level of nesting. For example, in the (KL)E

[1] Jones et al. (2004), Kilian (2008), Hamilton (2008) provide excellent surveys of the literature on the economic effects of oil shocks.

specification, capital and labour are combined first. The resulting composite is then combined with energy at the upper level of nesting. Which of the three specifications is more appropriate for modelling energy in production activities is subject to debate. Nine out of ten studies on climate policy listed by van der Werf (2008) either combine energy with a capital-labour composite or include all three inputs at the same level of aggregation. However, energy is first combined with capital in a widely used GTAP-E model developed by Burniaux and Truong (2002).[2] Many business cycle models that incorporate the energy input use the (KE)L specification, following the work of Kim and Loungani (1992).[3] Yet, the (KL)E specification is also common (Carlstrom and Fuerst, 2006; Alpanda and Peralta-Alva, 2010; Bodenstein et al., 2011).

The values of the elasticities of substitution and the choice of the input nesting structure can have a significant impact on theoretical predictions and economic inferences. For example, Vinals (1984) finds that the elasticity of substitution between energy and capital is a key parameter that determines the sign of the output response after an energy price increase. Jacoby et al. (2006) argue that the elasticity of substitution between energy and the capital-labour composite is critical in determining the costs of environmental policy. Small variations in the elasticity of substitution in energy, capital and labour inputs generates substantial differences in the growth rates in Schubert and Turnovsky (2011). Lecca et al. (2011) establish that theoretical results of a computable general equilibrium model can be rather sensitive to the choice of the input nesting in the production function.

Theoretical modellers often adopt a Cobb-Douglas production function, with a unitary elasticity of substitution between inputs (e.g. Goulder and Schneider, 1999; Popp, 2004, Finn, 2000). While this functional form is convenient, its empirical applicability has been challenged. Some recent studies find that empirical data do not exhibit

[2]The GTAP-E introduces energy substitution possibilities into the GTAP model of the Global Trade Analysis Project. Beckman and Hertel (2010) discuss the use and the validity of the model.

[3]Backus and Crucini (2000), de Miguel and Manzano (2006), Dhawan et al. (2010).

properties that are compatible with the standard neo-classical growth model that features the Cobb-Douglas production function with technological change (Klump et al., 2007). Others argue that restrictive assumptions on the nature of technological change, embedded in the Cobb-Douglas production function, can lead to an estimation bias in favour of this function (Antràs, 2004). Elasticity estimates based on flexible costs functions may also be problematic for applied theoretical research. For example, Monte Carlo experiments point out that the estimates of a trans-log function (Christensen et al., 1971) may be unreliable when the true value of the elasticity of substitution is not close to unity.[4]

In this study, we estimate the parameters of the CES function for all three possible nesting structures of capital, labour and energy, allowing for the possibility of factor-specific or input-neutral technological change. Our estimation is based on a system of three equations with cross-equations restrictions that are derived from cost minimization. The empirical model allows us to identify the elasticities of substitution at the lower and the upper level of the CES nesting, and to test the nature of technological change. We also test the validity of the cost minimization restrictions, to determine which production structure fits the data best.

Our study offers two improvements to the previous work. First, many empirical estimations of the elasticities of substitution between energy and other inputs are conducted using linear or non-linear single-equation models.[5] However, estimation of single-equation factor demand function could be systematically biased (David and van de Klundert, 1965; Willman, 2002). Our system estimation should help to address this bias. Second, many previous empirical studies are carried out with specific assumptions on the nesting of the CES production function or the nature of technological change (e.g. Sue Wing, 2003; Bosetti et al., 2006). However, the choice of the production

[4] Guilkey and Lovell (1980), Guilkey et al. (1983), Despotakis (1986), Prywes (1986).

[5] See, for example, Prywes (1986), Chang (1994), Khan (1994), Kemfert (1998), Kemfert and Heinz (2000), Kuper and van Soest (2003), Markandya and Suzette (2007).

structure may affect the value and the significance of the estimated elasticities between factors in the same pair. We do not impose any prior assumptions on the validity of the CES specification. Instead, we conduct tests and provide parameter estimates for all three nesting structures.

To the best of our knowledge, Kemfert (1998) and van der Werf (2008) are the only other studies that directly compare three different input combinations within a CES production function with technological change. Kemfert estimates production parameters based on aggregate and sectoral data of the German economy using a non-linear single equation model. Van der Werf works with an unbalanced panel of seven industries of the OECD countries. Due to data limitations, he provides estimates only at country or industry level. Industry level estimates by countries would be valuable for multi-sector general equilibrium models that incorporate energy and international trade, such as Beausejour et al. (1995), and Bohringer and Rutherford (1997).

In this study, we use an empirical model developed by van der Werf (2008). Our sample coverage and estimation techniques differ. We focus on ten Canadian manufacturing industries for the period 1962-1997, and estimate seemingly unrelated regressions for each industry. Canada provides an interesting case study. It is a small open economy, which exports both energy and traditional manufacturing goods. The design of an appropriate energy and climate change policy is at the center of policy and political debate in this country. Additional recent estimates of the elasticities of substitution between energy and other inputs would contribute to resolving the debate. Our results for the Canadian industries favour the K(LE) nesting structure, in terms of respecting theoretical restrictions imposed by cost minimization. We find some support for the Leontief and the Cobb-Douglas technologies, as well as limited evidence of exogenous technological change.

2. Theoretical and empirical framework

Our model specification is derived from three assumptions: (i) a nested CES production function, (ii) price-taking behaviour of firms on input markets and (iii) cost minimization. In our empirical estimation, we consider three possible ways of aggregating three inputs into a CES function: (KL)E, (KE)L and K(LE). In explaining our empirical methodology, we focus on the first case for concreteness. Theoretical and empirical equations for the other two cases are derived in a similar way.

In the (KL)E production structure, capital K and labour L are first combined in a CES composite Z, with the elasticity of substitution $\sigma_{K,L}$,

$$Z(t) = \left\{ \varphi \left[A_K(t) K(t) \right]^{\frac{\sigma_{K,L}-1}{\sigma_{K,L}}} + (1-\varphi) \left[A_L(t) L(t) \right]^{\frac{\sigma_{K,L}-1}{\sigma_{K,L}}} \right\}^{\frac{\sigma_{K,L}}{\sigma_{K,L}-1}}. \qquad (1)$$

Even though we work with the industry-level data, we do not index variables and parameters by industry in this section for notational ease. The composite Z is further combined with the energy input E, with the elasticity of substitution $\sigma_{KL,E}$,

$$Y(t) = \left\{ \phi \left[A_E(t) E(t) \right]^{\frac{\sigma_{KL,E}-1}{\sigma_{KL,E}}} + (1-\phi) Z(t)^{\frac{\sigma_{KL,E}-1}{\sigma_{KL,E}}} \right\}^{\frac{\sigma_{KL,E}}{\sigma_{KL,E}-1}}. \qquad (2)$$

The share parameters $\varphi \in (0,1)$ and $\phi \in (0,1)$. Production is possibly subject to factor-specific deterministic technological change, represented by $A_j(t) = (1+a_j)^t$, $j \in \{K, L, E\}$. Technological change is Hicks neutral with respect to all inputs if and only if $a_K = a_L = a_E$. In this case, we can factor the common values of technological change out of the brackets in (1) and (2). If $A_j(t)$, $j \in \{K, L, E\}$ grow at the different rates, technological change is defined as factor-specific.

The two-level CES production function implies a constant elasticity of substitution between the inputs at the lower level of nesting.[6] This is not generally the case for the

[6] The direct partial elasticity of substitution between two inputs x_i and x_j is defined as $\sigma_{ij} = -\partial \ln(x_i/x_j)/\partial \ln(p_i/p_j)$, where output and all other inputs except x_i and x_j are held constant.

elasticity of substitution between the other two pairs of inputs. The latter depend on the relative shares of inputs and the relative shares of composites in total costs, which typically vary over time. For the case of the (KL)E specification, the elasticity of substitution between capital and labour $\sigma_{K,L}$ is constant. The direct partial elasticities of substitution between these inputs and energy are harmonic means of $\sigma_{K,L}$ and $\sigma_{KL,E}$ (Sato, 1967). All partial elasticities of substitutions are constant only when all three inputs can be included in the same nesting. Then $\sigma_K = \sigma_{KL,E} = \sigma$, and the production function can re-written as

$$Y(t) = \left\{ \psi_1 \left[A_E(t) E(t)\right]^{\frac{\sigma-1}{\sigma}} + \psi_2 \left[A_K(t) K(t)\right]^{\frac{\sigma-1}{\sigma}} + \psi_3 \left[A_L(t) L(t)\right]^{\frac{\sigma-1}{\sigma}} \right\}^{\frac{\sigma}{\sigma-1}}, \quad (3)$$

where $\psi_1 = \phi$, $\psi_2 = (1-\phi)\varphi$, $\psi_3 = (1-\phi)(1-\varphi)$. Another special case occurs if $\sigma_{K,L} = 1$ or $\sigma_{KL,E} = 1$. The CES functions take Cobb-Douglas forms

$$Z(t) = \left[A_K(t) K(t)\right]^{\varphi} \left[A_L(t) L(t)\right]^{(1-\varphi)}, \text{ if } \sigma_{K,L} = 1, \quad (4)$$

$$Y(t) = \left[A_E(t) E(t)\right]^{\phi} Z(t)^{(1-\phi)}, \text{ if } \sigma_{KL,E} = 1. \quad (5)$$

Clearly, different assumptions about input nesting in the CES function imply different elasticities of substitution. Our study provides guidance on a possible range of values these elasticities can take, as well as on the specification of the production function that fits the Canadian data best.

We assume that firms minimize costs, taken input prices as given. The strong separability of the production function allows sequential decision making by a firm. The firm can first determine optimal demand at the upper level of nesting, then find the optimal levels of the components of the composite index. Based on the optimality conditions, van der Werf (2008) derives a system of three equations, which serves as a basis of our empirical estimation. For the (KL)E specification, the system of the theoretical equations is

$$e(t) - y(t) = a_E(\sigma_{KL,E} - 1) + \sigma_{KL,E}[p_Y(t) - p_E(t)], \tag{6}$$

$$\theta_{K,Z}(t) = a_K(\sigma_{K,L} - 1) + \frac{\sigma_{K,L} - 1}{1 - \sigma_{KL,E}}\theta_{Z,Y}(t) + (1 - \sigma_{K,L})[p_K(t) - p_Y(t)], \tag{7}$$

$$\theta_{L,Z}(t) = a_L(\sigma_{K,L} - 1) + \frac{\sigma_{K,L} - 1}{1 - \sigma_{KL,E}}\theta_{Z,Y}(t) + (1 - \sigma_{K,L})[p_L(t) - p_Y(t)], \tag{8}$$

All variables with small letters represent percentage changes, approximated by the first differences in logarithms. For example, $e(t) = \Delta \ln E(t) = \ln E(t) - \ln E(t-1)$. The variables $\theta_{N,M}(t) = \Delta \ln \Theta_{N,M}(t)$, where $\Theta_{N,M}(t)$ represents the cost share of N in the costs of producing M in period t.

Theoretical optimality conditions (6)-(8) will not hold exactly in the data, due to approximations and simplifications of the model and possible measurement errors. For the empirical estimation, we augment each theoretical equation with a stochastic error term $\varepsilon_j(t)$, $j = 1, 2, 3$, $E[\varepsilon_j(t)] = 0$. We assume that the error terms $\varepsilon'(t) = [\varepsilon_1(t), \varepsilon_2(t), \varepsilon_3(t)]$ are uncorrelated over time, but possibly correlated across the equations. For the (KL)E specification, we estimate the elasticities of substitution and the rates of technological change from the following system of three equations:

$$y_1(t) = a_E(\sigma_{KL,E} - 1) + \sigma_{KL,E}x_1(t) + \varepsilon_1(t), \tag{9}$$

$$y_2(t) = a_K(\sigma_{K,L} - 1) - \frac{1 - \sigma_{K,L}}{1 - \sigma_{KL,E}}x_{21}(t) + (1 - \sigma_{K,L})x_{22}(t) + \varepsilon_2(t), \tag{10}$$

$$y_3(t) = a_L(\sigma_{K,L} - 1) - \frac{1 - \sigma_{K,L}}{1 - \sigma_{KL,E}}x_{21}(t) + (1 - \sigma_{K,L})x_{32}(t) + \varepsilon_3(t). \tag{11}$$

The dependent variables are $y_1(t) = e(t) - y(t)$, $y_2(t) = \theta_{K,Z}(t)$, and $y_3(t) = \theta_{L,Z}(t)$. The explanatory variables are the percentage changes of the relative prices and of the cost shares: $x_1(t) = p_Y(t) - p_E(t)$, $x_{22}(t) = p_K(t) - p_Y(t)$, $x_{32}(t) = p_L(t) - p_Y(t)$, and $x_{21}(t) = \theta_{Z,Y}(t)$.

The empirical system (9)-(11) has non-linear cross-equation restrictions on the parameters of the model. We estimate this system using the non-linear seemingly unrelated regression technique in EViews. Given the parameter estimates, we test for the Cobb-Douglas functional form ($\sigma_{K,L} = 1$, $\sigma_{KL,E} = 1$) and for the non-nested CES function ($\sigma_{K,L} = \sigma_{KL,E}$). We also test whether technological change is input-neutral or factor specific. In our framework, exogenous technological change is input-neutral if the growth rates at the lower level of nesting are zeros but the growth rate at the upper level of nesting is different from zero. For the (KL)E specification, input-neutrality of technological change corresponds to a hypothesis that $a_K = a_L = 0$ and $a_E \neq 0$.

To assess which of the three possible nesting structures of the CES production function is more favoured by the data, we also estimate an unrestricted system of equations for each CES specification:

$$y_1(t) = \alpha_1 + \beta_1 x_1(t) + \varepsilon_1(t), \tag{12}$$

$$y_2(t) = \alpha_2 + \beta_{21} x_{21}(t) + \beta_{22} x_{22}(t) + \varepsilon_2(t), \tag{13}$$

$$y_3(t) = \alpha_3 + \beta_{31} x_{21}(t) + \beta_{32} x_{32}(t) + \varepsilon_3(t). \tag{14}$$

We test two theoretical restrictions implied by cost minimization: $\beta_{21} = \beta_{31}$ and $\beta_{22} = \beta_{32}$. We report the production function estimates even for the cases when theoretical restrictions are rejected. Our goal is to provide the parameter values for the CES functions that can be directly used in applied theoretical work. In this respect, our approach and motivation are similar to the analysis of van der Werf (2008). They are also consistent with many empirical studies of demand equations (see section 14.3 in Greene, 2003).

3. Data

We use industry-level data obtained from the productivity program database of Statistics Canada. The database provides nominal series for energy costs as well as the series for capital and labour compensation. It also contains chained-weighted prices and quantities of capital, labour and energy inputs. Statistics Canada does not directly report output at the KLE-level. We construct such measures of industrial output and the corresponding prices as Törnqvist aggregates of the inputs.

Our industry set comprises ten industries. Four industries are classified as energy-intensive manufacturing industries: primary metal, paper and allied products, chemical and chemical products, and non-metallic mineral products. The remaining six industries include transportation, food, transportation equipment, fabricated metal products, textile products, and machinery, except electrical machinery. Our industry choice is determined by the data availability. The data are annual, from 1962 to 1997. For each industry and for each input nesting, we estimate five regression parameters and six entries of the variance-covariance matrix of the error terms in (9)-(11), based on the sample of 36 observations.

Table 1 summarizes the average cost shares of capital, labour and energy inputs, as well as the average cost shares of input composites in total production costs. The time series of these shares are used in the empirical estimation. For given elasticities of substitution, the values of the share parameters φ and ϕ in the CES functions (1) and (2) can be calibrated from the average shares in Table 1.

Prior to estimating the system of the empirical equations, we checked all required relative prices, cost shares and input-to-output ratios (in percentage changes) for stationary. We could not reject the absence of a unit root in each series, based on the augmented Dickey-Fuller tests. The results of the tests are available upon request.

4. Results

In discussing the results, we first examine the tests of theoretical cost minimization restrictions. Then we present the estimates of the substitution elasticities and of the rates of technological change.

4.1. Cost-minimization restrictions

How energy, capital and labour enter the production function at the industry level is an unresolved empirical issue. While the use of the (KL)E and the (KE)L specifications is pervasive among theoretical modellers, there is no consensus on which of the two specifications, if any, has stronger empirical support. Our comparison of the adequacy of the three possible CES nesting structures is based on the tests of the theoretical restrictions $\beta_{21} = \beta_{31}$ and $\beta_{22} = \beta_{32.}$ Table 2 summarizes the results of individual Wald tests for the parameter equality.

The Canadian data favour the K(LE) specification, in which labour and energy are combined first. Both theoretical restrictions cannot be rejected in eight out of ten industries at the 5% significance level. For the remaining two industries (fabricated metal and paper and allied products), only one restriction can be rejected. By contrast, the (KE)L specification has the least empirical support, in terms of the number of the accepted theoretical restrictions. The restriction $\beta_{21} = \beta_{31}$ holds only for the primary metal industry. The other restriction ($\beta_{22} = \beta_{32}$) cannot be rejected in four industries: chemicals, food, machinery, and paper and allied products. For the (KL)E specification, both cost-minimization restrictions are satisfied for fabricated metal and machinery industries at the 5% significance level,

In comparison, van der Werf (2008) rejects the cross-equation restriction $\beta_{22} = \beta_{32}$ for all countries and all industries for the (KL)E and K(LE) nesting structures, and for 12 out of 19 cases for the (KE)L specification. Van der Werf uses the adjusted R^2 statistics of the panel regressions to compare the empirical models. Based on this statistic, the (KE)L specification is inferior to the other two production structures. The goodness of fit measures for the system of equations have to be generally treated with a caution. The comparison of the results for three possible CES nesting is further affected by the fact that the empirical systems have different independent and explanatory variables.

For the German economy, Kemfert (1998) prefers the (KE)L specification at the aggregate level and (KL)E specification for five out of seven industries. The R^2 statistics reported in her study are meaningful since the estimation is based on a single equation with the same variables. Interestingly, the R^2 statistics for the K(LE) specification in Kemfert (1998) are very close to the ones for the preferred specifications, albeit the differences between the values are not tested statistically.

4.2. Elasticities of substitution

Table 3 reports the estimated elasticities of substitution and their standard errors for all three possible nestings of the CES production function. Tables 4 provides the probability values for the restrictions of the unitary values of the elasticities (Cobb-Douglas production function) and for the hypotheses that capital, labour and energy have the same elasticity of substitution (the production function takes a single-level CES form). Several conclusions emerge from our results.

First, there is a substantial variation of the elasticities across the industries at the lower and the higher levels of nesting. The estimates fall in the range between 0.07 and 1.00. We find no stark differences between energy-intensive and energy non-intensive industries.

Second, the elasticities of substitution are generally positive. The only exceptions are the food and machinery industries. We cannot reject perfect complementarity (Leontief technology) between capital and the composite of labour and energy ($\sigma_{LE,K} = 0$) in the K(LE) specification for the food industry at the conventional levels of significance. For the machinery industry, perfect complementarity between energy input and the composite of capital and labour ($\sigma_{KL,E} = 0$) can be rejected at the 5% significance level, but not at the 10% level.

Third, capital, labour and energy are generally not substitutable with the same ease. Table 4 indicates that the hypothesis that the elasticities of substitution between the three inputs are equal can be accepted only in five cases: for primary metals, transportation equipment and textile products for the (KL)E specification, and non-metallic mineral products and machinery for the (KE)L case. Note, however, that for these five industries at least one of the restrictions $\beta_{21} = \beta_{31}$ or $\beta_{22} = \beta_{32}$ is rejected, as the statistics in Table 2 imply.

Fourth, the elasticities of substitution at the upper level of nesting tend to be smaller than at the lower level of nesting. For example, the values $\sigma_{KE,L}$ and $\sigma_{LE,K}$ are on average more than 2 and 4.5 smaller than the values $\sigma_{K,E}$ and $\sigma_{L,E}$. The property is expected a-priori in the aggregation of several inputs in production. As Sato (1967) argues, the input aggregation is justified by the similarity of "techno-economic characteristics," one of which is the ease of substitutability. Only in three cases (chemicals, transportation and food), the upper nesting elasticity $\sigma_{KL,E}$ exceeds the lower nesting elasticity $\sigma_{K,L}$ for the (KL)E specification.

Finally, Canadian firms tend to find it most difficult to substitute between capital and labour. The estimated elasticities $\sigma_{K,L}$ are all below 0.5 for the (KL)E specification. The relative low values of the elasticities $\sigma_{KE,L}$ and $\sigma_{LE,K}$ at the upper nesting also imply low substitutability between labour and capital. For every period of the sample, and for every CES nesting, we computed the implied partial elasticities of substitution between the pairs of inputs from the lower and the upper nesting. The average elasticities between the capital and labour were the lowest among the three elasticities for the (KE)L and the K(LE) specifications. By contrast, the largest estimates are for the elasticities of substitution between labour and energy $\sigma_{L,E}$. These values vary between 0.60 and 1.00. In four cases (primary metals, paper, transportation equipment and machinery), we cannot reject the Cobb-Douglas function between labour and energy at the conventional levels of significance.

4.3. Technological change

Table 5 reports the estimates of the growth rates a_K, a_L and a_E for the three specifications of the production function for each industry. For six industries in our sample, we find no evidence of the exogenous technological change, either neutral or factor-specific. We cannot reject that all growth rate parameters are zero in all three specifications of production functions at the 5% significance level in the following industries: fabricated metal, machinery, non-metallic mineral products, primary metal, textile and transportation.

For the remaining four industries (chemicals, food, paper and transportations) we test whether technological change is factor-specific or input-neutral. Table 6 reports the results. Recall that technological change is input neutral if all productivity terms in (1) and (2) grow at the same rate, which can be factored out. For each specification of the production function in Table 6, the first column gives the probability value for the Wald tests that both growth factors at the lower level of nesting are equal to zero. A small p-value indicates a rejection of the hypothesis that technological change is input-neutral. The second column reproduces the value of the growth factor at the upper level of nesting. We cannot reject the null that technological change is input-neutral in five cases at the 5% level of significance: paper and transportation industries for the (KL)E specification; chemicals, food and paper for the (KE)L specification; and paper for the K(LE) specification. It should be noted, however, that in none of these five cases both theoretical restrictions of cost minimization are satisfied (see Table 2).

The K(LE) specification for the chemicals industry is the only entry in Table 6, for which both theoretical restrictions $\beta_{21} = \beta_{31}$ and $\beta_{22} = \beta_{32}$ hold. Our results suggest that this industry was subject to labour-, capital- and energy-augmenting technological change of 2%, -1.5% and -7.8% per year between 1962 and 1997.

5. Concluding remarks

We estimated the parameters of nested CES functions with capital, labour and energy inputs and exogenous technological change, based on the industry-level Canadian data. Our estimates of the elasticities of substitution can be directly used in applied general equilibrium models with multiple sectors that incorporate energy. Since our estimation is derived from cost minimization, the use of the estimates can help address criticisms of weak empirical foundations of computable general equilibrium models (Beckman and Hertel, 2010), and the paucity of industry-level estimates.

In terms of respecting theoretical restrictions imposed by cost minimization, the Canadian data favour the K(LE) specification, in which energy and labour are combined first using a CES function, and then this energy-labour composite is combined with capital in a second CES function. We also find that capital and labour have rather low, and often the lowest, elasticity of substitution. Both results are in contrast with the assumptions of existing theoretical studies. While the examples of models using the (KL)E or the (KE)L specifications are abundant, we have not come across any study that adopts the K(LE) structure. It remains to be seen how theoretical inferences regarding the role of energy in the economy are affected by the K(LE) specification and the values of the elasticities of substitution between energy and other inputs.

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Table 1: Average cost shares in total expenditures (1962-1997)

Industry/Shares	$\Theta_{K,Y}$	$\Theta_{L,Y}$	$\Theta_{E,Y}$	$\Theta_{KL,Y}$	$\Theta_{KE,Y}$	$\Theta_{LE,Y}$
Chemicals	0.425	0.457	0.119	0.881	0.543	0.575
	(0.071)	(0.082)	(0.047)	(0.047)	(0.082)	(0.071)
Fabricated metal	0.282	0.692	0.025	0.975	0.308	0.718
	(0.029)	(0.028)	(0.007)	(0.007)	(0.028)	(0.029)
Food	0.372	0.585	0.043	0.957	0.415	0.628
	(0.049)	(0.050)	0.007)	(0.007)	(0.050)	(0.049)
Machinery	0.298	0.684	0.018	0.982	0.316	0.702
	(0.044)	(0.045)	0.005)	(0.005)	(0.045)	(0.044)
Non-metal. mineral products	0.345	0.546	0.109	0.891	0.454	0.655
	(0.042)	(0.031)	0.026)	(0.026)	(0.031)	(0.042)
Paper and allied products	0.320	0.539	0.141	0.859	0.461	0.680
	(0.098)	(0.078)	0.042)	(0.042)	(0.078)	(0.098)
Primary metal	0.234	0.580	0.186	0.814	0.420	0.766
	(0.078)	(0.054)	0.042)	(0.042)	(0.054)	(0.078)
Textile	0.279	0.684	0.036	0.964	0.316	0.721
	(0.036)	(0.036)	0.012)	(0.012)	(0.036)	(0.036)
Transportation	0.247	0.607	0.146	0.854	0.393	0.753
	(0.038)	(0.023)	0.037)	(0.037)	(0.023)	(0.038)
Transportation equipment	0.284	0.692	0.024	0.976	0.308	0.716
	(0.069)	(0.067)	(0.005)	(0.005)	(0.067)	(0.069)

Notes: The values in parantheses are standard errors. The shares do not necessarily add up to one due to rounding. Each value $\Theta_{N,Y}$ represents the cost share of N in the costs of producing output Y.

Table 2: Tests of cost minimization restrictions

Industry / *p*-values	(KL)E		(KE)L		K(LE)	
	$\beta_{21}=\beta_{31}$	$\beta_{22}=\beta_{32}$	$\beta_{21}=\beta_{31}$	$\beta_{22}=\beta_{32}$	$\beta_{21}=\beta_{31}$	$\beta_{22}=\beta_{32}$
Chemicals	0.190	0.012	0.000	0.282	0.864	0.504
Fabricated metal	0.270	0.818	0.005	0.015	0.044	0.225
Food	0.000	0.000	0.003	0.758	0.880	0.879
Machinery	0.069	0.081	0.000	0.998	0.055	0.563
Non-metal. mineral prod.	0.165	0.038	0.000	0.003	0.331	0.612
Paper and allied products	0.769	0.000	0.001	0.671	0.904	0.020
Primary metal	0.041	0.000	0.454	0.000	0.282	0.162
Textile	0.544	0.000	0.000	0.002	0.822	0.284
Transportation	0.380	0.001	0.000	0.005	0.796	0.414
Transportation equipment	0.020	0.000	0.000	0.001	0.227	0.208
Industries with the accepted restrictions	7	2	1	4	9	9

Notes: The table reports the p-values for the Wald tests. A *p*-value below 0.05 indicates that the null of the parameter equality can be rejected at the 5% significance level. The last row gives the number of industries for which a theoretical restriction cannot be rejected at the 5% significance level.

Table 3: Estimated elasticities of substitution

Industry/ elasticities	(KL)E		(KE)L		K(LE)	
	$\sigma_{KL,E}$	$\sigma_{K,L}$	$\sigma_{KE,L}$	$\sigma_{K,E}$	$\sigma_{LE,K}$	$\sigma_{L,E}$
Chemicals	0.4833*** (0.0406)	0.2514*** (0.0301)	0.2739*** (0.0446)	0.4315*** (0.0521)	0.1485*** (0.0369)	0.6492*** (0.0324)
Fabricated metal	0.2436*** (0.0899)	0.4567*** (0.0284)	0.3144*** (0.0327)	0.4384*** (0.0478)	0.4037*** (0.0492)	0.7803*** (0.0604)
Food	0.7435*** (0.0427)	0.3522*** (0.0364)	0.2900*** (0.0468)	0.6621*** (0.0630)	0.1028 (0.0664)	0.7463*** (0.0313)
Machinery	0.2042* (0.1068)	0.4650*** (0.0217)	0.3714*** (0.0277)	0.4401*** (0.0431)	0.3771*** (0.0515)	0.9519*** (0.0445)
Non-metal. mineral products	0.3061*** (0.0699)	0.4395*** (0.0177)	0.3582*** (0.0297)	0.4667*** (0.0413)	0.4247*** (0.0257)	0.6631*** (0.0524)
Paper etc.	0.1529*** (0.0286)	0.4389*** (0.0102)	0.0868*** (0.0187)	0.4743*** (0.0171)	0.1281*** (0.0463)	1.0008*** (0.0038)
Primary metal	0.1026*** (0.0285)	0.1549*** (0.0151)	0.0858*** (0.0160)	0.3034*** (0.0164)	0.0707** (0.0317)	0.9969*** (0.0017)
Textile	0.3397*** (0.0673)	0.3678*** (0.0301)	0.2272*** (0.0297)	0.4598*** (0.0387)	0.2611*** (0.0698)	0.9008*** (0.0333)
Transportation	0.5409*** (0.0500)	0.3605*** (0.0317)	0.1692*** (0.0407)	0.4475*** (0.0471)	0.3712*** (0.0372)	0.6094*** (0.0362)
Transp. equipment	0.3560*** (0.0590)	0.3860*** (0.0282)	0.2662*** (0.0320)	0.4328*** (0.0302)	0.2630*** (0.0705)	0.9754*** (0.0151)

Notes: The values in parantheses are standard errors. The stars ***, ** and * indicate the significance level of 1%, 5% and 10%.

Table 4: Tests for Cobb-Douglas function (unitary elasticities) and non-nested CES function (common elasticities)

	(KL)E			**(KE)L**			**K(LE)**		
Industry / *p*-values	$\sigma_{KL,E}$=1	$\sigma_{K,L}$=1	$\sigma_{KL,E}$= $\sigma_{K,L}$	$\sigma_{KE,L}$=1	$\sigma_{K,E}$=1	$\sigma_{KE,L}$= $\sigma_{K,E}$	$\sigma_{LE,K}$=1	$\sigma_{L,E}$=1	$\sigma_{LE,K}$= $\sigma_{L,E}$
Chemicals	0.00	0.00	0.00	0.00	0.00	0.04	0.00	0.00	0.00
Fabricated metal	0.00	0.00	0.01	0.00	0.00	0.02	0.00	0.00	0.00
Food	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Machinery	0.00	0.00	0.01	0.00	0.00	0.21	0.00	0.28	0.00
Non-metal. mineral prod.	0.00	0.00	0.04	0.00	0.00	0.06	0.00	0.00	0.00
Paper etc.	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.84	0.00
Primary metal	0.00	0.00	0.11	0.00	0.00	0.00	0.00	0.07	0.00
Textile	0.00	0.00	0.68	0.00	0.00	0.00	0.00	0.00	0.00
Transportation	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Transp. equip.	0.00	0.00	0.61	0.00	0.00	0.00	0.00	0.10	0.00

Notes: The Table reports probability values for the two-sided Wald tests. A *p*-value below 0.05 indicates that the null of the parameter equality can be rejected at the 5% significance level.

Table 5: Rates of factor-specific technological change

Industry/rate of technlogical change	Capital a_K (KL)E	Capital a_K (KE)L	Capital a_K K(LE)	Labor a_L (KL)E	Labor a_L (KE)L	Labor a_L K(LE)	Energy a_E (KL)E	Energy a_E (KE)L	Energy a_E K(LE)
Chemicals	-0.0187**	-0.0195*	-0.0147*	0.0128***	0.0168***	0.0200***	-0.0429*	-0.0427	-0.0778*
	(0.0089)	(0.0107)	(0.0082)	(0.0038)	(0.0055)	(0.0058)	(0.0226)	(0.0267)	(0.0406)
Fabricated metal	-0.0092	-0.0069	-0.0027	-0.0011	0.0028	-0.0004	-0.0117	-0.0171	-0.0433
	(0.0168)	(0.0133)	(0.0114)	(0.0057)	(0.0038)	(0.0091)	(0.0107)	(0.0184)	(0.0467)
Food	-0.0233***	-0.0237*	-0.0192***	0.0100***	0.0086***	0.0082**	0.0022	0.0155	-0.0130
	(0.0082)	(0.0127)	(0.0056)	(0.0031)	(0.0033)	(0.0038)	(0.0267)	(0.0108)	(0.0376)
Machinery	-0.0232	-0.0198	-0.0146	0.0031	0.0067	0.0036	-0.0091	-0.0099	-0.0910
	(0.0207)	(0.0180)	(0.0140)	(0.0054)	(0.0047)	(0.0295)	(0.0154)	(0.0267)	(0.3069)
Non-metal. mineral products	-0.0044	-0.0035	-0.0019	0.0013	0.0025	0.0019	-0.0079	-0.0085	-0.0139
	(0.0092)	(0.0086)	(0.0080)	(0.0038)	(0.0045)	(0.0055)	(0.0116)	(0.0189)	(0.0302)
Paper etc.	-0.0179	-0.0153	-0.0220**	0.0142	0.0127	-0.0480	-0.0182	-0.0271*	0.1809
	(0.0643)	(0.0546)	(0.0089)	(0.0204)	(0.0092)	(0.2436)	(0.0118)	(0.0144)	(0.9165)
Primary metal	-0.0180	-0.0163	-0.0150	0.0048	0.0061	0.0895	-0.0167	-0.0196	-2.7878
	(0.0450)	(0.0558)	(0.0108)	(0.0376)	(0.0113)	(0.9922)	(0.0106)	(0.0285)	(3.3198)
Textile	-0.0275	-0.0226	-0.0140	-0.0021	0.0046	0.0026	-0.0234*	-0.0266	-0.1641
	(0.0219)	(0.0184)	(0.0138)	(0.0102)	(0.0060)	(0.0188)	(0.0128)	(0.0198)	(0.1227)
Transportation	-0.0076	-0.0069	-0.0069	0.0057	0.0056**	0.0070*	-0.0274**	-0.0241**	-0.0334**
	(0.0099)	(0.0097)	(0.0078)	(0.1276)	(0.0025)	(0.0037)	(0.0140)	(0.0118)	(0.0165)
Transp. equipment	-0.0398	0.0190	-0.0280	0.0057	0.0093	0.0131	-0.0107	-0.0097	-0.3134
	(0.0336)	(0.0245)	(0.0234)	(0.0093)	(0.0068)	(0.0317)	(0.0128)	(0.0213)	(0.4414)

Notes: The values in parantheses are standard errors. The stars ***, ** and * indicate the significance level of 1%, 5% and 10%.

Table 6: Input-neutral versus factor-specific technological change

Industry/ test results	(KL)E		(KE)L		K(LE)	
	$a_K = a_L = 0$ (*p*-value)	a_E	$a_K = a_E = 0$ (*p*-value)	a_L	$a_L = a_E = 0$ (*p*-value)	a_K
Chemicals	0.0007	-0.0429*	0.0871	0.0168***	0.0027	-0.0147*
Food	0.0001	0	0.0626	0.0086***	0.0227	-0.0192***
Paper etc.	0.1204	0	0.0558	0	0.9807	-0.0220**
Transport.	0.1612	-0.0274**	0.0219	0.0056**	0.0374	0

Notes: For each specification of the production function, the first column reports the probability value for the Wald tests that both growth factors at the lower level of nesting are equal to zero. The second column reproduces the value of the growth factor at the upper level of nesting from Table 5. The stars ***, ** and * indicate the significance level of 1%, 5% and 10%.