CAHIER DE RECHERCHE #1213E
Département de science économique
Faculté des sciences sociales
Université d'Ottawa

WORKING PAPER #1213E
Department of Economics
Faculty of Social Sciences
University of Ottawa

On the Measurement of Indignation

Paul Makdissi* and Myra Yazbeck†

September 2012

* CIRPÉE and Department of Economics, University of Ottawa, 55 Laurier E. (10125), Ottawa, Ontario, Canada, K1N 6N5; Email: paul.makdissi@uottawa.ca.
† CIRPÉE and Department of Epidemiology Biostatistics and Occupational Health, Purvis Hall, McGill University, Montreal, Canada, H3A 1A2; Email: myra.yazbeck@mcgill.ca

Abstract

Recently, a lot of attention is given to income variations occurring at the top of the income distribution. "What happens to the top 1%?" is a question of crucial importance on the political level (Occupy Wall Street Movement) as well as on income inequality measurement level. Despite this increased interest, there is no rigorous measurement framework available in the literature for the measurement of "indignation". To fill this gap, this paper proposes a simple framework for the measurement of indignation. It exposes the ethical principles underlying an indignation index and develops restricted positional dominance conditions that produce robust orderings of indignation between income distributions. It also proposes a parametric class of indignation indices that may be used to produce complete orderings when the restricted positional dominance tests do not lead to satisfactory orderings. Finally, the paper offers a brief empirical illustration using the World Top Incomes Database.

Key words: *Indignation, Inequality, Positional Dominance, Lorenz Curve*
JEL Classification: I39

Résumé

Récemment, beaucoup d'attention a été accordé aux variations de revenu au sommet de la distribution des revenus. "Que se passe-t-il avec les 1% des revenus les plus élevés?" est devenu une question d'importance tant du point de vue politique (Mouvement Occupy Wall Street) que du point de vue de la mesure des inégalités de revenu. Malgré cet intérêt récent, il n'existe pas dans la littérature de cadre de mesure rigoureux pour la mesure de l' "indignation". Cet article propose de combler cette lacune en proposant un cadre simple pour la mesure de l'indignation. Les principes éthiques sous-jacents à un indice d'indignation sont énoncés et les conditions de dominance positionnelle sont développés afin de permettre de produire des ordres robustes de distribution de revenus en terme d'indignation. L'article propose aussi une classe d'indices paramétriques afin de produire des ordres complets en absence de dominance. Finalement, l'article présente une illustration empirique à l'aide du World Top Incomes Database.

Mots clés: *Indignation, Inégalité, Dominance positionnelle, Courbe de Lorenz*
Classification JEL: I39

1 Introduction

The international financial crisis of 2007-2009 and the food price crisis affected the vast majority of the world's population and increased polarization (see Stiglitz, 2012). As a result the global political landscape witnessed many episodes of social unrest. The trigger point was the Arab Spring that resulted in the ousting of many dictators.[1] Inspired by these social disapprovals, Greece and Spain protested against austerity measures that were imposed to cope with the consequences of the increase in public dept resulting from the rescue of the banking system. This movement, the *indignados* (indignants), has in turn influenced the "Occupy Wall Street" movement that became an international protest movement against social and economic inequality. This protest movement has its well-known slogan, "We are the 99%", which refers to the increase in the income share of the top 1% of the income distribution.

This paper is motivated by this chain of events as well as the recent interest in the analysis of the top of the income distribution. There is a growing body of empirical literature that analyzes the top of the income distribution. In a seminal paper, Piketty (2003) presents a complete picture of the income distribution (including the top of the distribution) using French fiscal data . A series of subsequent works conduct the same analysis for the United States, the UK, Canada, Germany and Switzerland as well as Australia (see Piketty and Saez ,2003; Atkinson, 2005; Saez and Vaell, 2005; Fortin, Green, Lemieux, Milligan and Riddell, 2012; Bach, Corneo and Steiner, 2009; Dell, 2005 and Atkinson and Leigh, 2007). Further developments on the historical evolution of the top of the income distribution were provided in Piketty and Saez (2006) and Atkinson, Piketty and Saez (2011).[2]

Despite the recent and growing interest in the top of the income distribution, the literature is still limited to empirical analyses that focus on the proportion of total income held by individuals at the top of the income distribution. There is only one paper that tackles measurement issues that

[1]More specifically, Ben Ali in Tunisia, Mubarak in Egypt and Gaddafi in Lybia.

[2]See Atkinson and Piketty (2007) and Leigh (2009) for a complete literature review on this topic.

are related to the top of the income distribution. It consists of a theoretical investigation of the link between the Gini index and top income shares when the top proportion of income analyzed is infinitesimal (Alvaredo, 2011). To our best knowledge, apart from the proportion of income held by the individuals in the top of the income distribution, no formal or specific index was developed for the measurement of indignation.[3] While the use of the proportion of income held by the top $x\%$ of individuals is informative, it provides an incomplete picture of the structure of this portion of the income distribution. More specifically, the proportion of income is insensitive to the redistribution of income between individuals located at the top of the distribution.[4] To avoid this problem, a common practice in the current empirical consists of computing this proportion at many top quantiles (*e.g.*, 10%, 5%, 1%, 0.5%,0.01%, 0.005%, 0.0001%).

The normative measurement literature requires that formal indices satisfy a set of normative axioms that are clearly defined. Such formalized indices that measure indignation are obviously still missing from this literature. The objective of this paper is to fill this gap in the literature by proposing a simple framework for the measurement of indignation. To do so, we first present the ethical principles that the indices should obey, then develop restricted positional dominance conditions that produce robust orderings of indignation between income distributions. We show that the initial version of the Lorenz curve (see Lorenz, 1905) that ranks individuals from the richest to the poorest is more appropriate, when establishing these partial orderings, than the usual Lorenz curve commonly used in the income distribution literature. We also propose a parametric class of indignation indices that may be used to produce complete orderings when the restricted positional dominance tests do not lead to satisfactory orderings.

The remaining of the paper unfolds as follows. The next section present the theoretical framework. Section 3 develops restricted positional dominance results based on the initial version of the Lorenz curve. Section 4 presents a parametric class of indignation indices. Finally, Section 5 offers a

[3]An indignation index will give a synthetic picture of the structure of the income distribution for the top $x\%$ of the income distribution.

[4]Just like the headcount index is in the case of poverty measurement.

brief empirical illustration based the World Top Incomes Database that was developed by Facundo Alvaredo, Tony Atkinson, Thomas Piketty and Emmanuel Saez.

2 A Measurement Framework

Let y represent income and F represents its cumulative distribution. For expositional ease, we define the relative income x as $x = y/\mu$, $\mu = \int_0^\infty ydF(y)$. Define $\Phi(p) = \inf\{x : 1 - F(\mu x) \leq p\}$ where $p \in [0, 1]$ represents the socioeconomic rank of individuals when they are ranked from the richest to the poorest. Let ω be the portion of the top of the income distribution that leads to indignation. In this context, an index of indignation may be expressed as:

$$I(\Phi, \omega) = \int_0^1 \nu(p, \omega)\Phi(p)dp. \tag{1}$$

where $\nu(p, \omega)$ represents the weight attached to an individual at the top $100p\%$ of the distribution. The usual way to summarize the top of the income distribution in the literature until now is to use the top income share index, $S(\Phi, \omega)$. This index gives the share of total income that is held by the top $100\omega\%$ of the income distribution. It is a particular case of equation (1) where the following specific discontinuous weight function is used:

$$v(p, \omega) = \begin{cases} 1, & \text{if} \quad p \leq \omega \\ 0, & \text{if} \quad p > \omega \end{cases}. \tag{2}$$

In order to be more transparent with respect to ethical judgement, it is convenient to lay out some axioms that describe the normative views on indignation. There are two axioms that should be obeyed by any indignation indices:

Axiom 1 ***(Focus)*** *Given two income distributions where the relative incomes at the top* $100\omega\%$ *of the income distribution are the same in both cases, the indignation index measured on either distribution should give the same value.*

In order to obey Axiom 1, an indignation index must have a weight function such that $\nu(p, \omega) = 0$ for all $p \in (\omega, 1]$.

Axiom 2 ***(Monotonicity)*** *Given other things, an increase in the relative income of an individual in the top* $100\omega\%$ *of the income distribution must increase the indignation index.*

In order to obey Axiom 2, an indignation index must have a weight function such that $\nu(p,\omega) \geq 0$ for all $p \in [0,\omega]$ and $\nu(p,\omega) > 0$ for some finite interval between 0 and ω.

There are other normative principles drawn from the income distribution literature that may be useful for indignation measurement purposes: (a) the well known Pigou-Dalton transfer principle, (b) positional transfer sensitivity, and (c) generalized positional transfer principles.

Axiom 3 ***(Transfer)*** *Given other things, a pure transfer of income from an individual in the top* $100\omega\%$ *of the income distribution to any other individual that is poorer must decrease the indignation index.*

To satisfy Axiom 3, the indignation index should have a weight function such that for $i = 1$, $\nu^{(i)}(p,\omega) \leq 0$.[5]

In the literature on income distribution, it is not uncommon to impose more structure on indices. This is achieved by selecting transfer principles that the index should obey. There is a wide range of transfer principles. The Principle of Transfer Sensitivity (Kolm, 1976), is one of the most widely used principle in income distribution analysis. It postulates that an income transfer, valued to be δ, from a higher-income individual to a lower-income one yields a better impact on social welfare insofar as incomes (y) are the lowest possible. Hence, a decision maker respecting the Principle of Transfer Sensitivity prefers a transfer from y_3 to y_4 rather than from y_1 to y_2 (where $y_1 > y_2 > y_3 > y_4$ and $p_1 < p_2 < p_3 < p_4$), as long as the rank of the individuals remains unchanged after such transfers and that $y_1 - y_2 = y_3 - y_4$. Despite its wide applicability, there is no consensus on the desirability of this principle as it is insensitive to individuals' rank. To overcome this problem, a homologous principle was developed in a rank-dependent framework by Mehran (1976) and Kakwani (1980). It

[5] $\nu^{(i)}(p,\omega)$ represents the i-th derivative of $\nu(p,\omega)$ with respect to p.

assumes that an income transfer from a high-income individual to a low-income individual yields a better impact on social welfare, insofar as individuals' ranks are the highest possible (remember that, in our framework, we rank individuals from the richest to the poorest). A decision maker respecting the Principle of Positional Transfer Sensitivity prefers a transfer from y_3 to y_4 rather than from y_1 to y_2, given that the rank of the individuals remains unchanged after such transfers and that $p_2 - p_1 = p_4 - p_3$. In a more general perspective, Fishburn and Willig (1984) proposed a class of Generalized Transfer Principles that states that as the order of normative principle increases, the weight that is associated to transfers in the bottom of the income distribution increases. These principles were adjusted to fit the rank dependant context by Aaberge (2009). To adapt the formal generalization of these principles to the measurement of indignation, we follow closely Makdissi and Mussard (2008). Let $\Delta_{p,\gamma} I(\delta, \Phi, \omega)$ be the variation in the indignation index $I(\Phi, \omega)$ induced by a transfer δ from the person at rank p to the one at rank $p + \gamma < \omega$, $\gamma > 0$. Let us define $\Delta^{\sigma}_{p,\Gamma^{\sigma}} I(\delta, \Phi, \omega)$. This term is recursively deduced as follows:

$$\Delta^{2}_{p,\Gamma^{2}} I(\delta, \Phi, \omega) := \Delta_{p+\gamma_2,\gamma_1} I(\delta, \Phi, \omega) - \Delta_{p,\gamma_1} I(\delta, \Phi, \omega), \tag{3}$$

where $\Gamma^2 = (\gamma_1, \gamma_2)$, $\gamma_i > 0$,

$$\vdots$$

$$\Delta^{\sigma}_{p,\Gamma^{\sigma}} I(\delta, \Phi, \omega) := \Delta^{\sigma-1}_{p+\gamma_{\sigma},\Gamma^{\sigma-1}} I(\delta, \Phi, \omega) - \Delta^{\sigma-1}_{p,\Gamma^{\sigma-1}} I(\delta, \Phi, \omega), \tag{4}$$

where $\Gamma^{\sigma} = (\gamma_1, \gamma_2, \ldots, \gamma_{\sigma})$, $\Gamma^1 = \gamma_1$ and $\gamma_i > 0$.

Axiom 4 ***(sth-degree Positional Transfer Sensitivity)*** *An indignation index, $I(\Phi, \omega)$, satisfies the Principle of sth-degree Positional Transfer Sensitivity if, $\Delta^{s-2}_{p,\Gamma^{s-2}} I(\delta, \Phi, \omega) \geq \Delta^{s-2}_{p',\Gamma^{s-2}} I(\delta, \Phi, \omega)$ for all $p' < p < \omega$.*

The Positional Transfer Principle of order s requires that $(-1)^{(s)} \nu^{(s-1)}(p, \omega) \leq 0$.

3 Restricted Positional Dominance

This objective of this section is to develop restricted positional dominance condition that identifies partial robust orderings of income distribution in terms of indignation. The usual Lorenz curve commonly used in the economics literature plots the cumulative share of income against the cumulative share of total population when individuals are ranked from the poorest to the richest. However, the initial version of the Lorenz curve (Lorenz, 1905) plots the cumulative share of income against the cumulative share of total population when individuals are ranked from the richest to the poorest. For a reason that will become clearer as we proceed in this section, we adopt this initial version of the Lorenz curve as in Lorenz (1905). This initial Lorenz (Λ) curve is defined as follows:

$$\Lambda(p) = \int_0^p \Phi(s)ds. \tag{5}$$

It is important to note that the top income index, $S(\Phi, \omega)$, is equal to the point estimate of the initial Lorenz curve at rank ω.

In order to develop the restricted positional dominance tests that will identify robust indignation orderings, it is useful to define formally the initial Lorenz curves of order s:

$$\Lambda^s(p) = \begin{cases} \int_0^p \Lambda^{s-1}(s)ds, & \text{if} \quad s \in 3, 4, ... \\ \Lambda(p), & \text{if} \quad s = 2. \end{cases} \tag{6}$$

Let us define classes of indignation indices $\Upsilon^s(\omega)$, $s \in \{2, 3, ...\}$, as:

$$\Upsilon^s(\omega) := \left\{ I(\Phi, \omega) \;\middle|\; \begin{array}{c} \nu(p, \omega) \in \widehat{C}^{s-1}, \\ \nu(p, \omega) = 0 \ \forall p \in [\omega, 1] \\ (-1)^{i-1} \nu^{(i-1)}(p, \omega) \geq 0 \text{ for } i = 1, 2, ..., s \end{array} \right\}, \tag{7}$$

where $\nu^{(0)}(p, \omega) = \nu(p, \omega)$ and $\widehat{C}^s(\omega)$ represents the set of functions that are $(s-1)$ times differentiable almost everywhere on $[0, 1]$. Note here that the widely used top income share index, $S(\Phi, \omega)$, does not belong to $\Upsilon^2(\omega)$ since it fails to meet the continuity requirement over the interval $[0, 1]$.

Proposition 1 *All indices $I(\Phi, \omega) \in \Upsilon^2(\omega)$ obey the Focus Axiom, the Monotonicity Axiom and the Transfer Axiom.*

Proposition 2 *All indices $I(\Phi,\omega) \in \Upsilon^s(\omega)$, $s \in \{3,4,...\}$, obey the Focus Axiom, the Monotonicity Axiom, the Transfer Axiom and all the ith-degree Positional Transfer Sensitivity axioms of orders $i \in 3,...,s$.*

Using the initial Lorenz curves of order s, it is possible to identify robust indignation orderings for all indices that belong to $\Upsilon^s(\omega)$:

Theorem 1 *$I(\Phi_A,\omega) \geq I(\Phi_B,\omega)$ for all $I(\Phi,\omega) \in \Upsilon^s(\omega)$, $s \in \{2,3,...\}$, and all $\omega \leq \omega^+$ if and only if:*

$$\Lambda^s_A(p) \geq \Lambda^s_B(p)\ \forall p \in [0,\omega^+]. \tag{8}$$

Proof. To prove for sufficiency, we first need to integrate by parts equation (1):

$$\begin{aligned} I(\Phi,\omega) &= \int_0^1 \nu(p,\omega)\Phi(p)dp. && (9)\\ &= \nu(p,\omega)\Lambda^2(p)\big|_0^1 - \int_0^1 \nu^{(1)}(p,\omega)\Lambda^2(p)dp. && (10)\end{aligned}$$

Since by definition $\nu(1,\omega) = 0$ and $\Lambda^2(0) = 0$, the first term of the right hand side of equation (10) is equal to 0. This means that:

$$I(\Phi,\omega) = -\int_0^1 \nu^{(1)}(p,\omega)\Lambda^2(p)dp. \tag{11}$$

Assume that for some $s > 2$, we have:

$$I(\Phi,\omega) = (-1)^{s-2}\int_0^1 \nu^{(s-2)}(p,\omega)\Lambda^{s-1}(p)dp. \tag{12}$$

Integrating equation (12) by parts yields:

$$I(\Phi,\omega) = \nu^{(s-2)}(p,\omega)\Lambda^s(p)\Big|_0^1 + (-1)^{s-1}\int_0^1 \nu^{(s-1)}(p,\omega)\Lambda^s(p)dp. \tag{13}$$

Since by definition $\nu^{(s-2)}(1,\omega) = 0$ and $\Lambda^s(0) = 0$, the first term of the right hand side of equation (13) is again equal to 0. It follows that:

$$I(\Phi,\omega) = (-1)^{s-1}\int_0^1 \nu^{(s-1)}(p,\omega)\Lambda^s(p)dp. \tag{14}$$

Given that equation (11) obeys the relation depicted in equation (12), then, by induction, equation (14) is true for all integer $s \in \{2, 3, ...\}$. Using equation (14) it is possible to find an expression for a variation $\Delta I(\Phi_A, \Phi_B, \omega) = I(\Phi_A, \omega) - I(\Phi_B, \omega)$:

$$\Delta I(\Phi_A, \Phi_B, \omega) = (-1)^{s-1} \int_0^1 \nu^{(s-1)}(p, \omega) \left[\Lambda_A^s(p) - \Lambda_B^s(p)\right] dp. \tag{15}$$

From the definition of the set of indices $\Upsilon^s(\omega)$ we know that $(-1)^{s-1}\nu^{(s-1)} \geq 0$. Thus, if $[\Lambda_A^s(p) - \Lambda_B^s(p)] \geq 0$ for all $p \in [0, \omega^+]$, then $\Delta I(\Phi_A, \Phi_B, \omega) \geq 0$.

To prove necessity, consider the set of polynomial functions $I(\Phi, \omega) \in \Upsilon^s(\omega)$ for which the $(s-2)$th derivative of $\nu(p, \omega)$ is of the following form:

$$\nu^{s-2}(p, \omega) = \begin{cases} (-1)^{s-2}\varepsilon & \text{for } p \leq \overline{p} \\ (-1)^{s-2}(\overline{p} + \varepsilon - p) & \text{for } \overline{p} < p \leq \overline{p} + \varepsilon \\ 0 & \text{for } p > \overline{p} \end{cases}. \tag{16}$$

Since $\nu^{s-2}(p, \omega)$ is differentiable almost everywhere, it satisfies the conditions in (7). Thus, indignation indices whose weight function $\nu(p, \omega)$ have the particular above form for $\nu^{s-2}(p, \omega)$ belong to $\Upsilon(\omega)$. This implies that:

$$\nu^{s-1}(p, \omega) = \begin{cases} 0 & \text{for } p \leq \overline{p} \\ (-1)^{s-1} & \text{for } \overline{p} < p \leq \overline{p} + \varepsilon \\ 0 & \text{for } p > \overline{p} \end{cases}. \tag{17}$$

Suppose that $[\Lambda_A^s(p) - \Lambda_B^s(p)] < 0$ on an interval $[\overline{p}, \overline{p} + \varepsilon]$ for ε that can be arbitrarily close to 0. For $\nu(p, \omega)$ defined as in (16), expression (15) is then negative and $I(\Phi_A, \omega) < I(\Phi_B, \omega)$. Hence, it cannot be that $[\Lambda_A^s(p) - \Lambda_B^s(p)] < 0$ for $p \in [\overline{p}, \overline{p} + \varepsilon]$. ■

At this point, it is important to note that the top income share index, $S(\Phi, \omega)$ does not belong to Υ^2. However, since $S(\Phi, \omega) = \Lambda^2(\omega)$, we can state the following corollary

Corollary 1 *If $I(\Phi_A, \omega) \geq I(\Phi_B, \omega)$ for all $I(\Phi, \omega) \in \Upsilon^2(\omega)$, and all $\omega \leq \omega^+$, if and only if $S(\Phi_A, \omega) \leq S(\Phi_B, \omega)$ for all $\omega \in [0, \omega^+]$.*

Corollary 1 implies that even if the top income share index, $S(\Phi, \omega)$, does not belong to the set of indignation indices $\Upsilon^2(\omega)$, it is included in the robust rankings that will be obtained at the second order of restricted positional dominance. This is a very important subtlety as it relates to the most widely used indignation index in the literature.

4 A Parametric Class of Indices

The application of the restricted positional dominance conditions in Theorems 1 may not always produce complete orderings between all comparisons of income distributions. In this case, the use a parametric class of indices comes in handy when complete orderings are required. For this purpose, we propose the following parametric class:

$$I_\sigma(\Phi, \omega) = \int_0^\omega \left(\frac{\omega - p}{\omega} \right)^\sigma \Phi(p) dp, \tag{18}$$

where $\sigma \geq 0$ can be interpreted as a parameter of inequality aversion. It is important to note that, except for $\sigma = 0$, all indices $I_\sigma(\Phi, \omega)$ vary between $\frac{1}{2(1+\sigma)} \left[1 - (1 - \omega)^{(1+\sigma)} \right]$ (no indignation, i.e. a perfectly equal distribution) and 1 (perfect indignation). The index $I_0(\Phi, \omega)$ varies between ω (perfect equality) and 1 (perfect indignation).[6]

When $\sigma = 0$, $I_0(\Phi, \omega) = S(\Phi, \omega)$. Any other value for the inequality aversion parameter will imply inequality aversion for the distribution of income among the $100\omega\%$ richest.

Proposition 3 *When $\sigma = 0$, $I_\sigma(\Phi, \omega)$ satisfies the Focus and Monotonicity axioms. In addition, it satisfies the Transfer axiom if $\sigma > 0$ and all the principles of sth degree Positional Transfer Sensitivity, $s \in \{3, 4, ...\}$, if $\sigma > s - 2$.*

5 Illustration

In this section, we provide a brief illustration of the methodology proposed in this paper using the World Top Incomes Database that was developed by Facundo Alvaredo, Tony Atkinson, Thomas

[6] One may also use the index

$$\widetilde{I}_\sigma(\Phi, \omega) = \begin{cases} \frac{I_\sigma(\Phi,\omega) - \frac{1}{2(1+\sigma)}\left[1-(1-\omega)^{(1+\sigma)}\right]}{1 - \frac{1}{2(1+\sigma)}\left[1-(1-\omega)^{(1+\sigma)}\right]} \text{ for } \sigma > 0 & \text{for } \sigma > 0 \\ \frac{I_0(\Phi,\omega) - \omega}{1 - \omega} & \text{for } \sigma > 0 \end{cases}.$$

This index belongs to the $[0, 1]$ interval and produces exactly the same orderings of income distributions than $I_\sigma(\Phi, \omega)$.

Piketty and Emmanuel Saez.[7] In their paper, Atkinson, Piketty and Saez (2011) highlight three empirical regularities in the evolution of top incomes as summarized by the index $S(\Phi, 0.01)$ for all surveyed countries. More specifically, there is a fall in $S(\Phi, 0.01)$ between 1914 to 1945 followed by a relative stability of the index between 1945 and 1980 then an increase again in the beginning of the 80s. They also highlight that Anglo-Saxon economies experience a substantially higher increase in $S(\Phi, 0.01)$ compared to countries in continental Europe[8].

Given that empirical evidence suggests that Anglo-Saxon countries experience the most important increase in indignation, this paper will focus on the United States, the United Kingdom, Australia and Canada. To conduct comparisons between these countries we will take two points in time: 1982 and 2007.[9] Figure 1 displays the second order dominance test for indignation's comparisons for all four countries in two points in time (2007 and 1982). Based on Theorem 1, we can confirm that there is an unambiguous increase in indignation between 1982 and 2007 for all four countries. This result is in line with the findings of the papers surveyed in Atkinson, Piketty and Saez (2011) and extends its validity to any indignation threshold $\omega \in [0, 0.1]$ and any indignation indices that obey the Focus, Monotonicity and Transfer axioms. Furthermore, based on Corollary 1, we can affirm that this result is valid for the top income share index, $S(\Phi, \omega)$ for any indignation threshold $\omega \in [0, 0.1]$. Setting the indignation threshold at 1% (i.e., $\omega = 0.01$) is a common practice in this literature as it is believed that the dynamics of income at the top 1% of the distribution differs from that of the other 99%. Our empirical result reveals that the increase in indignation is not only valid for a 1% cutoff but is still observed for all cutoffs between 0 and 10%.

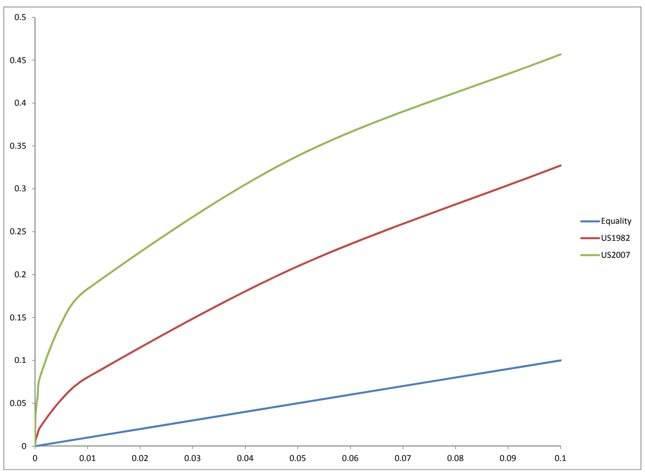
[7] This database can be found at http://g-mond.parisschoolofeconomics.eu/topincomes/. This website offers information at some quantiles $p_1 = 0.0001$, $p_2 = 0.0005$, $p_3 = 0.001$, $p_4 = 0.005$, $p_5 = 0.01$, $p_6 = 0.05$ and $p_7 = 0.1$. We have used this information to produce the results of this empirical illustration. Linear interpolation has been used to produce the Λ curves. For the indices, we have allowed a weight $\nu(p_i) = \int_{p_{i-1}}^{p_i} \left(\frac{\omega - p}{\omega}\right)^{\sigma} dp$ for the income share of individuals between p_{i-1} and p_i (with $p_0 = 0$).

[8] There is one interesting empirical finding in Saez and Veall (2005). They point to the fact that the French-speaking community in the province of Quebec, Canada, experience a lower increase in $S(\Phi, 0.01)$ than the rest of the Canadian population. The author hints that this may be caused by a language barrier to migration that decreases the menace of brain drain to the US.

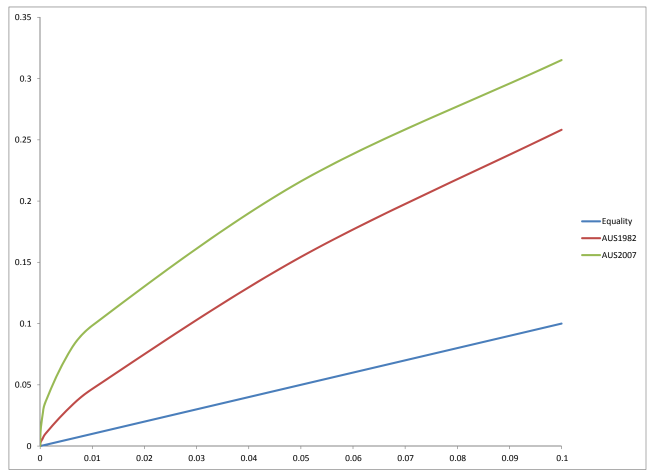
[9] We were constrained by the Canadian Longitudinal Administrative Data bank for which The World Top Income database has no information prior to 1982 and after 2007.

Figure 1: Comparisons of indignation between 2007 and 1982 (2^{nd} order of dominance)

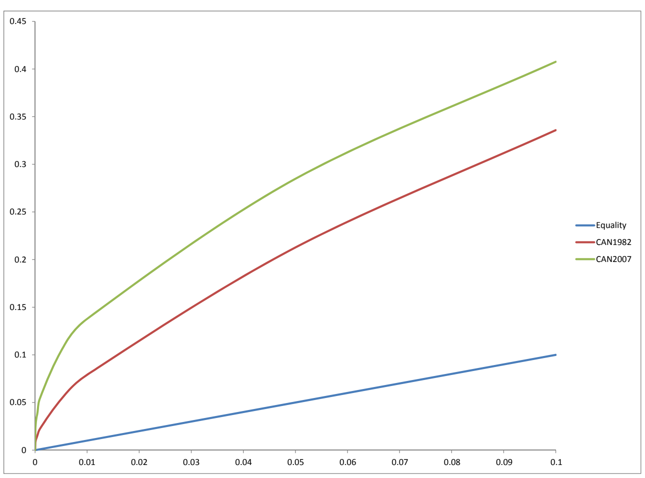
United States



Australia



Canada



United Kingdom

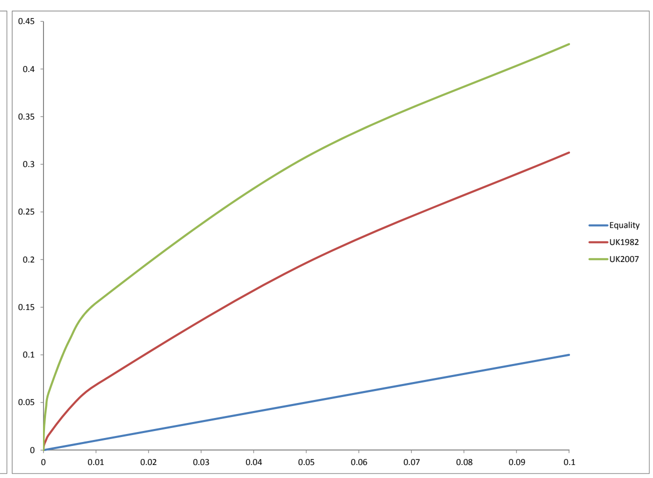


Figure 2: Comparisons of indignation between countries (2^{nd} order of dominance)

1982 2007

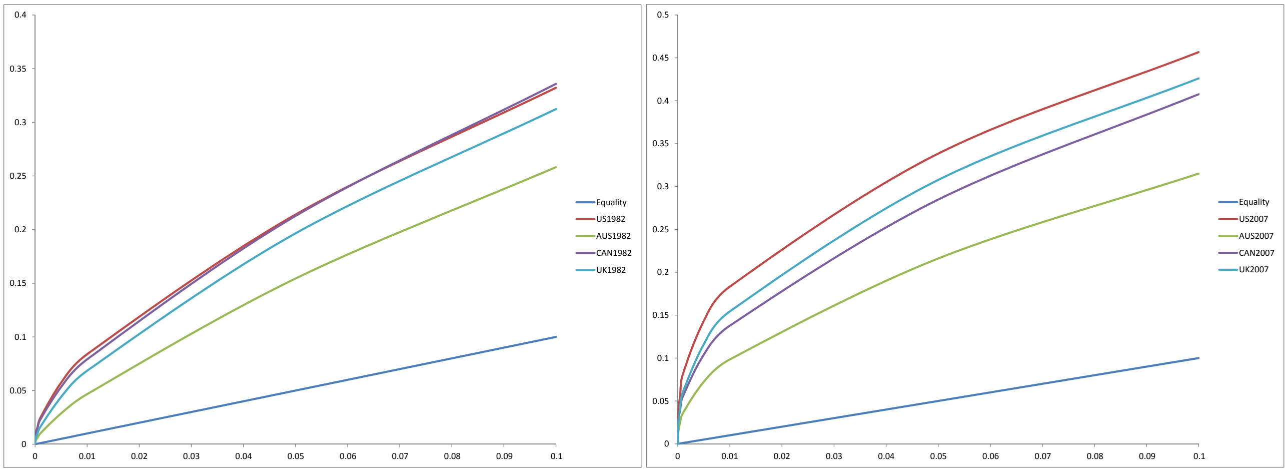


Figure 2 displays the second order dominance test for the comparisons of indignation using the same specification as earlier. It appears that, in 1982, there is more indignation in the United States and Canada than in the United Kingdom and Australia. Also, there is more indignation in the United Kingdom than in Australia. This ranking is valid for the top income share index, $S(\Phi, \omega)$ and for all indignation indices that obey the Focus, Monotonicity and Transfer axioms. It is also valid for all indignation thresholds $\omega \in [0, 0.1]$. However, the ranking between the United States and Canada is not robust even if we increase the order of dominance to 20.[10] As for the indignations' comparisons for the year of 2007, a complete ordering for all four countries is achieved with the United states ranking first, the United Kingdom second, Canada third and Australia last. This ordering is valid for the top income share index, $S(\Phi, \omega)$ and for all indignation indices that obey the Focus, Monotonicity and Transfer axioms. It is also valid for all indignation thresholds $\omega \in [0, 0.1]$. Comparing countries' orderings between 1982 and 2007 one can notice the change in the ranks for Canada and the United Kingdom: the United Kingdom having more indignation than

[10]To provide a robust ranking for these two countries one has to rely on a particular parametric form for the index. We will come back to this below.

Canada in 2007.

Figure 3: Comparisons of indignation between the United States in 1982 and Australia in 2007

2^{nd} order dominance 3^{rd} order dominance

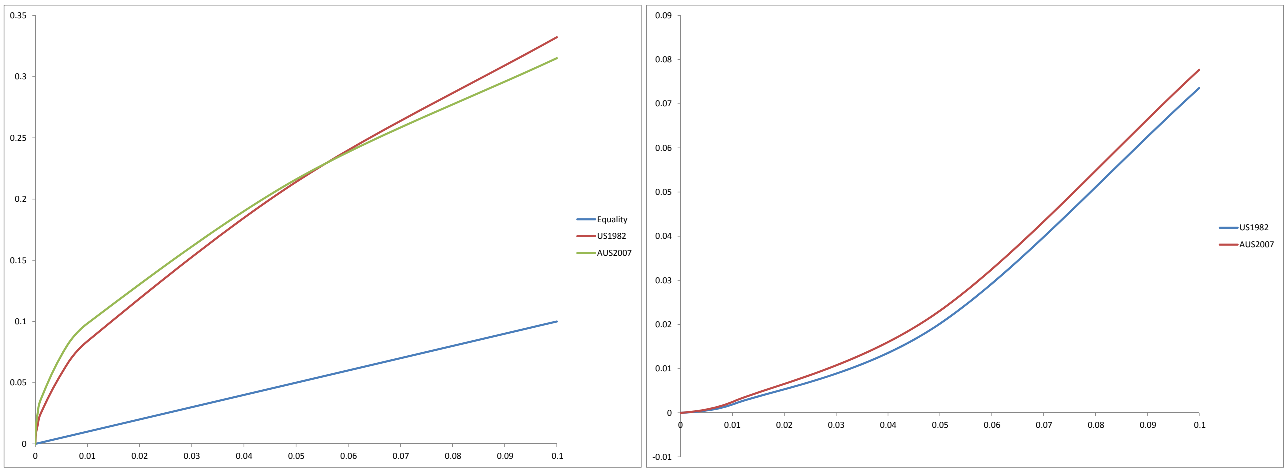


Figure 3 offers a comparison between the United States in 1982 (the country with the highest indignation in 1982) and Australia in 2007 (the country with the lowest indignation in 2007). A careful examination of the second order dominance test reveals a crossing between $\Lambda^2_{US1982}(\omega)$ and $\Lambda^2_{AUS2007}(\omega)$. This means that the ranking obtained is not be robust for all indignation thresholds $\omega \in [0, 0.1]$. However, the test produces a robust ranking for a smaller interval of indignation thresholds (i.e.,$\omega \in [0, 0.05]$). Therefore, it is safe to conclude that there is more indignation in Australia in 2007 than in the United States in 1982 and that this result is valid for the top income share index, $S(\Phi, \omega)$ and for all indignation indices that obey the Focus, Monotonicity and Transfer axioms and for all indignation thresholds $\omega \in [0, 0.05]$. To obtain a robust ranking for all indignation thresholds, one has to restrict the set of admissible indices by increasing the order of dominance. The second panel of Figure 3 displays the 3^{rd} order dominance test. This test produces a robust ranking: Australia in 2007 has more indignation than the United States in 1982. This result is valid for all indignation indices that obey the Focus, Monotonicity, Transfer and 3^{rd} degree Positional Transfer sensitivity axioms and for all indignation thresholds $\omega \in [0, 0.1]$.

Table 1: Estimated Indignation Indices

	1982			
	United States	Australia	Canada	United Kingdom
$I_0(\Phi, 0.01)$	0.0803	0.0467	0.0791	0.0685
$I_1(\Phi, 0.01)$	0.0001293	0.0000781	0.0001241	0.0001139
$I_2(\Phi, 0.01)$	0.0000821	0.0000482	0.0000783	0.0000712
$I_3(\Phi, 0.01)$	0.0000580	0.0000335	0.0000552	0.0000499
	2007			
	United States	Australia	Canada	United Kingdom
$I_0(\Phi, 0.01)$	0.1833	0.0984	0.1378	0.1544
$I_1(\Phi, 0.01)$	0.0002458	0.0001456	0.0001933	0.0002215
$I_2(\Phi, 0.01)$	0.0001628	0.0000947	0.0001262	0.0001450
$I_3(\Phi, 0.01)$	0.0001188	0.0000681	0.0000911	0.0001049

Table 1 displays the estimated values of the indignation indices $I_\sigma(\Phi, 0.01)$ in 1982 and 2007 for all four countries. While some of the information in this table may be tedious, given the robust rankings discussed earlier, it still provides additional ordering information regarding the ranking between the United States and Canada for the year of 1982. More specifically, for $I_\sigma(\Phi, 0.01)$ and for $s = 0$, 1, 2 and 3, there is less indignation in Canada than in the United States in 1982.

6 Conclusion

This paper attempts to fill a gap in the indignation measurement literature by developing an indignation index and a restricted positional dominance test based on the initial version of the Lorenz curve. The restricted positional dominance tests allow for the identification of indignation orderings that are robust for a wide spectra of ethical judgements. It also proposes a parametric class of indignation indices as well as an empirical illustration using data from the World Top Income Database for four Anglo-saxon countries. The results obtained corroborate an important empirical finding that can be found in the emerging literature on the top of income distributions: there is an increase of indignation in the last two decades in Anglo-Saxon countries. Within these countries, the ranking between the United Kingdom and Canada has changed between the periods 1982 and

2007; the United Kingdom having more indignation at the end of this period.

References

[1] Aaberge, R. (2009), Ranking Intersecting Lorenz Curves, *Social Choice and Welfare*, 33, 235-259.

[2] Alvaredo, F. (2011), A note on the relationship between top income shares and the Gini coefficient, *Economics Letters*, 110, 274-277.

[3] Atkinson, A.B. (2005), Top Incomes in the UK Over the Twentieth Century, *Journal of the Royal Statistical Society*, Series A, 168, 325-343.

[4] Atkinson, A.B. and A. Leigh (2007), The Distribution of Top Incomes in Australia, *Economic Record*, 83, 247-261.

[5] Atkinson, A.B. and T. Piketty (2007), *Top Incomes over the Twentieth Century: A Contrast Between Continental European and English Speaking Countries*, Oxford: Oxford University Press, 664-759.

[6] Atkinson, A.B., T. Piketty and E. Saez (2011), Top Incomes in the Long Run of History, *Journal of Economic Literature*, 49, 3-71.

[7] Bach, S., G. Corneo and V. Steiner (2009), From Bottom to Top: The Entire Income Distribution in Germany, 1992-2003, *Review of Income and Wealth*, 55, 303-330.

[8] Dell, F. (2005), Top Incomes in Germany and Switzerland Over the Twentieth Century, *Journal of the European Economic Association*, 3, 412-421.

[9] Fishburn, P.C. and R.D. Willig (1984),Transfer Principles in Income Redistribution, *Journal of Public Economics*, 25, 323-328.

[10] Fortin, N., D.A. Green, T. Lemieux, K. Milligan and W.C. Riddell (2012), Canadian Inequality: Recent Developments and Policy Options, *Canadian Public Policy*, 37, 121-145.

[11] Kakwani, N.C. (1980), On a Class of Poverty Measures, *Econometrica*, 48, 437-446.

[12] Kolm, J.-C. (1976), Unequal Inequlities I, *Journal of Economic Theory*, 12, 416-442.

[13] Leigh, A. (2009), Top Incomes, in W. Salverda, B. Nolan and T.M. Smeeding (eds.), *The Oxford Handbook of Economic Inequality*, New York: Oxford University Press, 150-176.

[14] Lorenz, M.O. (1905), Methods of Measuring the Concentration of Wealth, *Publications of the American Statistical Association*, 9, 209-219.

[15] Makdissi, P. and S. Mussard (2008), Analyzing the impact of indirect tax reforms on rank dependant social welfare functions: a positional dominance approach, *Social Choice and Welfare*, 30, 385-399.

[16] Mehran, F. (1976), Linear Measures of Income Inequality, *Econometrica*, 44, 805-809.

[17] Piketty, T. (2003), Income Inequality in France, 1901-1998, *Journal of Political Economy*, 111, 1004-1042.

[18] Piketty, T. and E. Saez (2003), Income Inequality in the United States, 1913-1998, *Quarterly Journal of Economics*, 118, 1-39.

[19] Piketty, T. and E. Saez (2006), The Evolution of Top Incomes: A Historical and International Perspective, *American Economic Review, Papers and Proceedings*, 96, 200-205.

[20] Saez, E. and M.R. Veall (2005), The Evolution of High Incomes in Northern America: Lessons from Canadian Evidence, *American Economic Review*, 95, 831-849.

[21] Stiglitz, J.E. (2012), *The Price of Inequality. How Today's Divided Society Endengers Our Future*, New York: W.W. Norton & Company.

[22] Zoli, C, (1999), Intersecting generalized Lorenz curves and the Gini index, *Social Choice and Welfare*, 16, 183-196.