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News versus Sunspot Shocks in Linear Rational Expectations Models

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Abstract

This paper compares news and sunspot shocks as sources of exogenous changes in beliefs by analyzing equilibria of rational expectations models. The similarities and differences between the two shocks are illustrated in a New Keynesian monetary model.

Keywords: *News Shocks; Sunspots; Expectation shocks*
JEL classification*:* C63; E32; E47; E52

Résumé

Ce papier compare les chocs de nouvelles et de tâches solaires en tant que sources des changements exogènes dans les croyances en analysant les équilibres des modèles avec anticipations rationnelles. Les similarités et les différences entre les deux chocs sont illustrées dans un modèle monétaire néo-keynésien.

Mots clés: *Nouvelles; Tâches solaires; Chocs d'anticipation*
Classification JEL: C63; E32; E47; E52

1. Introduction

How important are exogenous changes in agents' beliefs as a source of aggregate fluctuations? It is impossible to answer this question without a precise definition of such changes. In rational expectations models, exogenous changes in beliefs are represented in two conceptually different ways: through news and sunspot shocks. Up to date, effects of these shocks have been analyzed separately. This paper makes the first step towards understanding the similarities and differences between these two expectation shocks.

Sunspot shocks represent 'animal spirits' or unexplained waves of optimism and pessimism. Implications of sunspot shocks for business cycle dynamics have been studied, for example, by Farmer and Guo (1994), Benhabib and Farmer (1999), Lubik and Schorfheide (2004) and Hirose (2007). News shocks represent information that helps to predict future economic fundamentals but does not affect current and past fundamentals. The importance of news shocks for explaining business cycle fluctuations has been examined, for example, by Beaudry and Portier (2005, 2006, 2007), Jaimovich and Rebelo (2006) and Christiano, Ilut, Motto, and Rostagno (2007). This paper establishes that sunspot and news shocks generally impose distinct restrictions on impact responses and autocovariance properties of equilibria.

A complete set of solutions of linear rational expectations (LRE) models with expectation shocks is obtained with the method developed by Lubik and Schorfheide (2003). A simple new Keynesian monetary business cycle model is used to illustrate the results. The model exhibits indeterminacy and admits sunspot shocks if a central bank, following an interest rate rule, is not aggressive enough towards inflation. Lubik and Schorfheide (2004) argue that indeterminacy and possibly the existence of sunspot shocks may be relevant for understanding the dynamics of U.S. output, inflation and interest rates in a pre-Volcker period of 1960-1979. The sunspot shocks in the model are compared with news shocks about future monetary policy. Introduction of news shocks about monetary policy is partly motivated by the results of Cochrane (1998). He finds that a decomposition of monetary policy shocks into unanticipated and anticipated components can have an important impact on measuring the empirical response of output to changes in monetary policy. Examination of analytical solutions and impulse responses of the model to sunspot and news shocks highlights the differences between the two types of expectation shocks.

The rest of the paper is organized as follows. Section 2 defines news and sunspot shocks and explains their relation to fundamental shocks. It also demonstrates how to apply the solution method proposed by Lubik and Schorfheide (2003) to a *LRE* model with news shocks. Section 3 works with a new Keynesian model. Section 4 concludes.

2. *LRE* Models with Expectation Shocks and Their Solutions

A *LRE* model can incorporate three types of stochastic disturbances: fundamental shocks, news shocks and sunspot shocks. *Fundamental shocks* represent exogenous stochastic changes in preferences and technologies. Examples include changes in tastes, aggregate productivity, government spending and monetary policy. Both current and expected future fundamental shocks may appear in the equilibrium equations of the model. In contrast, expectation shocks do not appear directly in the equilibrium equations. They influence endogenous variables only through agents' beliefs.

News shocks represent exogenous changes in beliefs about future realizations of fundamental shocks. They capture the idea that agents can learn values of future fundamental shocks in advance. Examples of news shocks may include announcements of future *R&D* strategies, forthcoming policy reforms or intentions of a central bank to change its policy target. *Sunspot shocks* represent exogenous changes in agents' beliefs due to extrinsic uncertainty. They may reflect self-fulfilling swings in consumers' and investors' moods or in their perceptions about the current state of the economy.

This section defines expectation shocks formally and discusses the solutions to models with these shocks. A starting point of the analysis is a linear rational expectations model in the form:

$$\underset{n\times n}{\tilde{\Gamma}_0}\underset{n\times 1}{\tilde{y}_t} = \underset{n\times n}{\tilde{\Gamma}_1}\underset{n\times 1}{\tilde{y}_{t-1}} + \underset{n\times l}{\tilde{\Psi}_0}\underset{l\times 1}{z_t} + \underset{n\times l}{\tilde{\Psi}_1}E_t\underset{l\times 1}{z_{t+1}} + \underset{n\times k}{\Pi}\underset{k\times 1}{\eta_t},\ t\geq 0 \tag{1}$$

where $\tilde{y}_t$ is a vector of endogenous variables, $\tilde{y}_{-1}$ is given, z_t denotes fundamental shocks, and $E_t z_{t+1}$ represent the agents' subjective beliefs. The beliefs are rational: they coincide with mathematical expectations conditional on the available information set. Endogenous expectation errors η_t, satisfying $E_t\eta_{t+1}=0$, correspond to changes in agents's beliefs about endogenous variables between two consecutive periods. They depend on both endogenous variables and their expectations, which are generally unknown before the model is solved.

Fundamental shocks are assumed to follow a covariance stationary and invertible process:

$$\begin{aligned}\underset{q\times 1}{\tilde{z}_{t+1}} &= \underset{q\times q}{A}\tilde{z}_t + \underset{q\times l}{B}\underset{l\times 1}{\varepsilon_{t+1}} \\ \underset{l\times 1}{z_t} &= \underset{l\times q}{C}\tilde{z}_t\end{aligned} \tag{2}$$

where $C = \begin{bmatrix} \underset{l\times l}{I} & \underset{l\times(q-l)}{\tilde{C}} \end{bmatrix}$, $B = \begin{bmatrix} \underset{l\times l}{I} & \underset{l\times(q-l)}{0} \end{bmatrix}'$, I is an identity matrix. The vector of states $\tilde{z}_t$ contains, at least, current values of z_t $(q\geq l)$. Each fundamental shock is driven by its own impulse. The impulses ε_t are a vector white noise process with zero mean. Current and past realization of impulses are observable, and the law of motion for fundamental shocks is assumed common knowledge. The vector $\tilde{z}_0$ summarizes initial conditions in period zero.

The existing literature follows two approaches to modelling news shocks. The first approach postulates the existence of exogenous signals s_t that are (i) uncorrelated with current and past realizations of fundamental impulses ε, but are (ii) correlated with some of their future values up to T periods ahead $(0<T<\infty)$. The agents try to predict future fundamentals, based on the observed signals. The outcome of the signal extraction problem is a vector $\xi_t^{\varepsilon} = \begin{bmatrix} \xi_t^{1\prime} & ... & \xi_t^{T\prime} \end{bmatrix}'$. Each element $\xi_t^j \equiv E_t \varepsilon_{t+j} = E(\varepsilon_{t+j}|s_t, s_{t-1}, ...)$ denotes the conditional forecast of ε_{t+j} made in period t about values of fundamental impulses $j \geq 1$ period ahead. Cochrane (1994), Beaudry and Portier (2004), Jaimovich and Rebelo (2006) follow this approach.

The second approach to modelling news shocks recognizes that it is not the signals themselves but their information content that matter for agents. Under the rational expectations hypothesis, the fundamental impulses can be written as the sum of one-step-ahead forecast revisions or information innovations:

$$\varepsilon_t = \mu_t^0 + \mu_{t-1}^1 + ... + \mu_{t-T}^T, \quad \mu_t^j \equiv E_t \varepsilon_{t+j} - E_{t-1} \varepsilon_{t+j}, \ j \geq 0 \tag{3}$$

The variable μ_t^j defines an update in the expected value of ε_{t+j} between periods $t-1$ and t. The rationality of expectations implies that μ_t^j constitutes a martingale difference sequence. The vector μ_t^0 represents the unexpected component of the fundamental impulse ε. Vectors μ_t^j $(j \geq 1)$ can be interpreted as news shocks. They affect the future fundamental impulses with a delay. This approach to modeling news shocks has been used by Evans (1992), King and Plosser (1984), and Christiano, Ilut, Motto, and Rostagno (2007).

News shocks change the agents' beliefs about future fundamentals. The conditional forecasts of future fundamental shocks in period t is $E_t [z_{t+i}] = CA^i \tilde{z}_t + \Sigma_{j=1}^i CA^{i-j} BE_t [\varepsilon_{t+j}]$. Without news shocks, expectations of future fundamental shocks are completely determined by the history of their realizations, since the future impulses are unpredictable. With news shocks, agents' beliefs can change independently of fundamentals.

Using the definition of exogenous information innovations μ_t, the LRE model (1) is reformulated as follows:

$$\Gamma_0 y_t = \Gamma_1 y_{t-1} + \Psi \mu_t + \Pi \eta_t,\ t \geq 0 \tag{4}$$

$$y_{-1} \text{ given}$$

where

$$y_t = \begin{bmatrix} \tilde{y}_t \\ \tilde{z}_t \\ \xi_t^{\varepsilon} \end{bmatrix},\ \mu_t = \begin{bmatrix} \mu_t^0 \\ \mu_t^1 \\ \vdots \\ \mu_t^T \end{bmatrix} \Gamma_0 = \begin{bmatrix} \underset{n\times n}{\tilde{\Gamma}_0} & -\left(\tilde{\Psi}_0 C + \tilde{\Psi}_1 CA\right) & -\tilde{\Psi}_1 CB & \underset{n\times l(T-1)}{0} \\ \underset{q\times n}{0} & \underset{q\times q}{I} & \underset{q\times l}{0} & \underset{q\times l(T-1)}{0} \\ \underset{l\times n}{0} & \underset{l\times q}{0} & \underset{l\times l}{I} & \underset{l\times l(T-1)}{0} \\ \underset{l(T-1)\times n}{0} & \underset{l(T-1)\times q}{0} & \underset{l(T-1)\times l}{0} & \underset{l(T-1)\times l(T-1)}{I} \end{bmatrix}$$

$$\Gamma_1 = \begin{bmatrix} \underset{n\times n}{\tilde{\Gamma}_1} & 0 & 0 & 0 \\ 0 & \underset{q\times l}{A} & \underset{q\times l}{B} & 0 \\ 0 & 0 & 0 & \underset{l(T-2)\times l(T-2)}{I} \\ 0 & 0 & 0 & \underset{l\times l(T-1)}{0} \end{bmatrix},\ \Psi = \begin{bmatrix} \underset{\tilde{n}\times k}{0} & 0 \\ \underset{q\times l}{B} & 0 \\ 0 & \underset{kT\times kT}{I} \end{bmatrix},\ \Pi = \begin{bmatrix} \underset{\tilde{n}\times f}{\tilde{\Pi}} \\ \underset{(q+kT)\times f}{0} \end{bmatrix}$$

The full set of solutions to this model is obtained with the method developed by Lubik and Schorfheide (2003). Specifically, the endogenous expectation errors η_t are a linear function of information innovations μ_t and sunspot shocks ζ_t :

$$\eta_t = \left(A_\mu + A_\zeta M\right)\mu_t + A_\zeta \zeta_t \tag{5}$$

The matrices A_μ and A_ζ consist of the structural parameters of the model, while the coefficients of M are unrestricted. The dimension of the reduced form sunspot shocks vector ζ_t is determined by the difference between the number of endogenous expectation errors and the number of stability restrictions in the model. A_ζ is empty when the solution is unique. The only restriction on the sunspot shocks is $E_t\zeta_{t+1} = 0$.

The solution for the endogenous expectation errors (5) has four important characteristics. First, both news and sunspot shocks change agents's beliefs. Second, non-fundamental revisions in expectations, due to sunspots, can arise only under indeterminacy. Third, in models with multiple equilibria, the dynamic effects of sunspot shocks are unique. In contrast, the dynamics of news shocks are restricted, but not determined without extra assumptions. Finally, news shocks enrich dynamic correlation properties of the endogenous variables, as agents generally start acting upon new information about the future immediately. The similarities and differences between news and sunspot shocks are further illustrated in the context of a New Keynesian monetary model.

3. A New Keynesian Model

This section introduces news shocks about monetary policy into a simple New Keynesian monetary model studied by Lubik and Schorfheide (2003). The model exhibits indeterminacy if a central bank is not aggressive enough towards inflation. The differences between news and sunspot shocks are illustrated analytically and graphically.

3.1. Model Description

The model describes a closed economy with monopolitistically competitive firms that produce output with variable labour input. Prices are sticky due to adjustment costs or infrequent Calvo-type price setting. Households smooth their consumption streams by purchasing nominal government bonds. The central bank affects the economy through an interest rate rule. For simplicity, the only fundamental source of uncertainty is a monetary policy shock. The model has become a standard benchmark in the monetary literature. More details can be found, for example, in Clarida, Gali, and Gertler (1999) and Woodford (2003).

The log-linearized reduced-form of the model consists of three equations: (i) the intertemporal Euler equation, governing optimal consumption allocations, (ii) the expectation Phillips curve and (iii) the monetary policy rule:

Euler equation	$E_t x_{t+1} + \sigma E_t \pi_{t+1} = x_t + \sigma R_t$
Phillips curve	$\beta E_t \pi_{t+1} = \pi_t - \kappa x_t$
Monetary policy rule	$R_t = \psi \pi_t + \varepsilon_t,\ \varepsilon_t \sim N\left(0, \sigma_\varepsilon^2\right)$

Aggregate output x_t, inflation π_t, nominal interest rate R_t are expressed as log-deviations from the unique non-stochastic steady state. The parameter $0 < \beta < 1$ is the discount factor, $\sigma > 0$ is the intertemporal elasticity of substitution, $\kappa > 0$ is related to the speed of price adjustment, and $\psi \geq 0$ measures the elasticity of the interest rate response to inflation. The variable ε_t is a monetary policy shock.

To incorporate news shocks, it is assumed that in every period the agents receive an exogenous noisy signal s_t about monetary policy impulse $f \geq 1$ periods ahead:

$$s_t = \varepsilon_{t+f} + e_t,\ e_t \sim N\left(0, \sigma_e^2\right) \tag{6}$$

The noise e_t is uncorrelated with the monetary policy impulses ε_t at all lags and leads. The signal can be thought of as an announcement of future policy intentions.

Based on the signals, the agents predict future monetary policy. The outcome of the signal extraction problem are the conditional forecasts $E_t \varepsilon_{t+j} = \phi s_{t+j-f}$, if $0 < j \leq f$ and $E_t \varepsilon_{t+j} = 0$, if $j > f$. The coefficient $\phi = \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_e^2}$ depends on the signal to noise ratio. If the signal is perfectly informative ($\sigma_e^2 = 0$ and $\phi = 1$), the future monetary policy shock is known in advance. The signal becomes less valuable as its information content falls ($\sigma_e^2 \to \infty$ and $\phi \to 1$).

The actual monetary policy shock ε_t is assumed to be observed in period t. Thus, the beliefs about the monetary policy shock are updated twice: in periods $t - f$ and t. The information innovations represent these updates. The monetary policy shock consists of an unexpected impulse μ_t^0 and a news shock μ_{t-f}^f: $\varepsilon_t = \mu_t^0 + \mu_{t-f}^f$. By definition, $\mu_t^f = E_t\varepsilon_{t+f} - E_{t-1}\varepsilon_{t+f} = \phi s_t$ and $\mu_t^0 \equiv \varepsilon_t - \phi s_{t-f}$.

Different realizations of s and ε capture various scenarios. In analyzing the model, three possibilities are considered: (i) an unexpected monetary expansion ($s_t = 0$ and $\varepsilon_{t+f} < 0$, or $\mu_t^f = 0$ and $\mu_{t+f}^0 < 0$); (ii) a realized news about future monetary expansion ($s_t < 0$ and $\varepsilon_{t+f} = \phi s_t$, or $\mu_t^f < 0$ and $\mu_{t+f}^0 = 0$); (iii) an unrealized news about future monetary expansion ($s_t < 0$ and $\varepsilon_{t+f} = 0$, or $\mu_t^f < 0$ and $\mu_{t+f}^0 = -\mu_t^f$).

3.2. Solution

To derive the analytical solution, the period of anticipations f is set to one. Following the steps described in Lubik and Schorfheide (2003) and in the previous section, the New Keynesian model is reduced to a three-dimensional LRE system:

$$\underbrace{\begin{bmatrix} 1 & \sigma & 0 \\ 0 & \beta & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\Gamma_0}\underbrace{\begin{bmatrix} \xi_t^x \\ \xi_t^\pi \\ \xi_t^\varepsilon \end{bmatrix}}_{\xi_t} = \underbrace{\begin{bmatrix} 1 & \sigma\psi & \sigma \\ -\kappa & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{\Gamma_1}\underbrace{\begin{bmatrix} \xi_{t-1}^x \\ \xi_{t-1}^\pi \\ \xi_{t-1}^\varepsilon \end{bmatrix}}_{\xi_{t-1}} + \underbrace{\begin{bmatrix} \sigma & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}}_{\Psi}\underbrace{\begin{bmatrix} \mu_t^0 \\ \mu_t^1 \end{bmatrix}}_{\mu_t} + \underbrace{\begin{bmatrix} 1 & \sigma\psi \\ -\kappa & 1 \\ 0 & 0 \end{bmatrix}}_{\Pi}\underbrace{\begin{bmatrix} \eta_t^x \\ \eta_t^\pi \end{bmatrix}}_{\eta_t}$$

In this system, $\xi_t^x \equiv E_t x_{t+1}$ and $\xi_t^\pi \equiv E_t\pi_{t+1}$ are the conditional expectations of output and inflation, $\eta_t^x = x_t - \xi_{t-1}^x$ and $\eta_t^\pi = \pi_t - \xi_{t-1}^\pi$ are the endogenous forecast errors, $\xi_t^\varepsilon \equiv E_t\varepsilon_{t+1} = \phi s_t$ represents the conditional expectation of a future monetary policy shock, μ_t^0 and μ_t^1 are the information updates. The details of all derivations for this section are provided in the accompanied Technical Appendix.

The stability properties of the model are determined by the eigenvalues of the matrix $\left(\Gamma_0^{-1}\Gamma_1\right)$. The stability properties are not affected by the presence of the news shocks. One eigenvalue is equal to zero ($\lambda_0 = 0$), and the other two eigenvalues coincide with the eigenvalues for the model without news

$$\lambda_1, \lambda_2 = \frac{1}{2}\left(1 + \frac{\kappa\sigma + 1}{\beta}\right) \mp \frac{1}{2}\sqrt{\left(\frac{\kappa\sigma + 1}{\beta} - 1\right)^2 + \frac{4\kappa\sigma}{\beta}(1 - \psi)}$$

As in Lubik and Schorfheide (2003), the stable solution is unique if $\psi > 1$. Otherwise, the model has one-dimensional indeterminacy and multiple stable solutions.

3.2.1. Solution Under Determinacy ($\psi > 1$)

If the inflation elasticity of the interest rate rule ψ is greater than one, the equilibrium is unique and given by

$$\begin{bmatrix} x_t \\ \pi_t \\ R_t \end{bmatrix} = \frac{-\sigma}{1 + \kappa\sigma\psi} \begin{bmatrix} 1 \\ \kappa \\ \frac{-1}{\sigma} \end{bmatrix} \underbrace{\left(\mu_t^0 + \mu_{t-1}^1\right)}_{\varepsilon_t} - \frac{\sigma}{(1 + \kappa\sigma\psi)^2} \begin{bmatrix} 1 + \kappa\sigma(1 - \beta\psi) \\ \kappa(1 + \beta + \kappa\sigma) \\ \psi\kappa(1 + \beta + \kappa\sigma) \end{bmatrix} \underbrace{\mu_t^1}_{E_t\varepsilon_{t+1}} \tag{7}$$

The expectation errors η_t are determined solely by the information innovations:

$$\eta_t = -\frac{\sigma}{1 + \kappa\sigma\psi} \begin{bmatrix} 1 & \frac{1+\kappa\sigma(1-\beta\psi)}{1+\kappa\sigma\psi} \\ \kappa & \kappa\frac{1+\beta+\kappa\sigma}{1+\kappa\sigma\psi} \end{bmatrix} \begin{bmatrix} \mu_t^0 \\ \mu_t^1 \end{bmatrix} \tag{8}$$

The responses of a model to an unexpected monetary expansion are well known: output and inflation increase, while the nominal interest rate falls. The parameter κ governs the inflation-output trade-off. The response of output exceeds the response of inflation when $\kappa < 1$. The endogeneity of the monetary policy rule implies that the interest rate falls by less than the size of the unexpected monetary policy shock. The adjustment is completed within one period. Figure 1 provides an illustration. The solid lines on the figure plot impulse responses to an unexpected interest rate cut of 25 basis points in period 1. The parameter values $\beta = 0.99$, $\kappa = 0.5$ and $\sigma = 1$ are from Lubik and Schorfheide (2003). A more active policy, associated with higher ψ, influences the magnitude, but not the direction of the responses to an unexpected policy shock.

News shocks generate more interesting dynamics. Beliefs about forthcoming monetary expansion trigger the agents' reactions before the actual policy change. Foreseeing future expansion, firms increase their prices. Positive inflation increases the nominal interest rate, due to the policy feedback rule. The degree of central bank's aversion to inflation ψ influences the behavior of the real interest rate and output. If ψ is sufficiently high, news about future expansion has a contractionary effect on output. This is because a stimulative effect of an expected lower real interest rate is offset by the negative effect of a contemporaneous increase in the real interest rate. When the actual policy shock is observed and the agents learn whether or not their beliefs were correct, an additional adjustment takes place.

News about future policy expansion is more likely to trigger a decline in output when the number of anticipation periods is larger. In this case, the importance of the contemporaneous increase in the real interest rate becomes stronger relative to its expected decline in the future. Figure 1 illustrates the dynamics of output, inflation and the interest rate in response to a realized and unrealized news shocks for $f = 3$ and two values of the policy coefficient $\psi = 1.05$ and $\psi = 2.19$. The impulse responses on Figure 1 to the news shocks correspond to a belief, formed in period 1 that in period 4 the interest rate will be cut by 25-basis points. For the dashed lines, this belief is followed by the interest rate cut, as anticipated (a realized news shock). For the dashed-dotted lines, the belief is followed by no change in policy (an unrealized news shock). Note that until period 4, when the actual policy shock is observed, the responses to the realized and unrealized news shocks coincide.

Both the analytical solution and impulse responses highlight four features. First, news shocks strengthen the endogenous propagation of the model, since an anticipation of future changes triggers a prolonged reaction of the agents. Second, news shocks can lead to fluctuations in output, inflation and interest rates without any actual policy changes. This is demonstrated by the responses to an unrealized news shock. Third, the policy responsiveness towards inflation affects the dynamics of endogenous variables both quantitatively and qualitatively. Finally, there are noticeable differences in responses of output, inflation and interest rate to an unexpected policy shock and news shocks.

3.2.2. Solution Under Indeterminacy ($0 \leq \psi < 1$)

If the inflation elasticity of the interest rate rule ψ is less than one, the equilibrium output, inflation and nominal interest rate are:

$$\begin{bmatrix} x_t \\ \pi_t \\ R_t \end{bmatrix} = \begin{bmatrix} 1+\frac{\kappa\sigma}{\beta} & \sigma\left(\psi-\frac{1}{\beta}\right) & 0 \\ -\frac{\kappa}{\beta} & \frac{1}{\beta} & 0 \\ -\psi\frac{\kappa}{\beta} & \psi\frac{1}{\beta} & 0 \end{bmatrix} \begin{bmatrix} x_{t-1} \\ \pi_{t-1} \\ R_{t-1} \end{bmatrix} + \begin{bmatrix} \sigma L \\ 0 \\ 1 \end{bmatrix} \underbrace{\left(\mu_t^0 + \mu_{t-1}^1\right)}_{\varepsilon_t} + \begin{bmatrix} \eta_t^x \\ \eta_t^\pi \\ \psi\eta_t^\pi \end{bmatrix} \tag{9}$$

where L denotes the lag operator. The endogenous expectation errors η_t are a linear function of the information innovations μ_t and a sunspot shock ζ_t :

$$\eta_t = \frac{\kappa\sigma}{d^2} \begin{bmatrix} -\kappa\lambda_2 & \frac{b\kappa\lambda_2}{2(1+\kappa\sigma\psi)} \\ \lambda_2 - 1 - \kappa\sigma\psi & -\frac{b(\lambda_2-1-\kappa\sigma\psi)}{2(1+\kappa\sigma\psi)} \end{bmatrix} \mu_t + \frac{1}{d} \begin{bmatrix} \lambda_2 - 1 - \kappa\sigma\psi \\ \kappa\lambda_2 \end{bmatrix} (M\mu_t + \zeta_t) \tag{10}$$

The elements of matrix M are unrestricted, $d = \sqrt{(\kappa\lambda_2)^2 + (\lambda_2 - 1 - \kappa\sigma\psi)^2}$, $b = -2(1+\beta+\kappa\sigma-\beta\lambda_2)$. It can be shown that $b < 0$ and $1+\kappa\sigma\psi < \lambda_2$ for any values of $\kappa \geq 0$, $\sigma \geq 0$ and $\psi < 1$. The model has multiple equilibria, which can be obtained by varying the coefficients of the matrix M and specifying the process for ζ_t, with $E_t\zeta_{t+1} = 0$. Under indeterminacy, the solution generally exhibits an endogenous persistence: idiosyncratic shocks lead to a prolonged responses of output, inflation and interest rates.

The dynamic responses to unexpected policy shock and sunspots in this model have been described by Lubik and Schorfheide (2003). For completeness, their analysis is reproduced here. Figure 2 plots impulse responses to a sunspot shock, as well as to an unexpected monetary policy expansion and to a realized news shock under different assumptions for the coefficients of the matrix M. The parameter $\psi = 0.95$ is from Lubik and Schorfheide (2003). The realized news shock corresponds to a belief, formed in period 1, that in period 2 the interest rate will be cut by 25-basis points. This belief is followed by the interest rate cut, as anticipated.

A positive sunspot shock ζ_t increases the expectation errors of output and inflation, and hence output and inflation themselves. This shock can be interpreted as an exogenous increase in the forecast of inflation or output. In the context of the model, it is impossible to separate the two interpretations. In this sense, ζ_t is only a reduced form sunspot shock. The optimistic forecasts, due to sunspots, are self-fulfilling. They are validated by the actual increase in inflation and output in the current and future periods. In response to high inflation, the central bank raises the nominal interest rate. However, the actual and expected real interest rate decline, which stimulates current output. The dotted lines on Figure 2 give the impulse responses to a sunspot shock.

In contrast to the sunspot shocks, the dynamic effects of an unexpected monetary policy shock and a news shock are not unique. Lubik and Schorfheide (2003) propose two possible benchmarks for selecting values of the matrix M. Under the 'orthogonality' assumption, $M = \begin{bmatrix} 0 & 0 \end{bmatrix}$, and the shocks μ_t and ζ_t have orthogonal impacts on the expectation errors η_t. In this case, the model predicts a decline in inflation in response to an unanticipated monetary expansion. The impulse responses of output, inflation and interest rate for this case are represented by the dashed-dotted lines on Figure 2.

Under the 'continuity' assumption, the coefficients of the matrix M are chosen so that the effects of μ_t on the endogenous expectation errors coincide for the cases of determinacy and indeterminacy. When $M = [\frac{\sigma}{d}\left(1 - \frac{\lambda_2\left(1+\kappa^2\right)}{1+\kappa\sigma\psi}\right), \frac{\sigma}{d}\frac{1}{1+\kappa\sigma\psi}(1 + \kappa\sigma\left(1-\beta\psi\right) - \frac{\lambda_2\left(\left(1+\kappa^2\right)(1+\kappa\sigma)+\kappa\beta(\kappa-\sigma\psi)\right)}{1+\kappa\sigma\psi})]$ the impact matrix of the information innovations on the endogenous expectation errors coincide with (8). This solution preserves qualitative properties of the model under determinacy and prevents abrupt changes in endogenous expectation formation, as the economy crosses the boundary that separates determinacy and indeterminacy regions. The impulse responses of output, inflation and interest rates for this case are represented by the dashed-dotted lines on Figure 2.

Under indeterminacy, both news and sunspots affect the endogenous expectation errors and beliefs. The critical distinction between the two shocks is the non-uniqueness of the impact responses to news shocks. By varying the coefficients of the matrix M, it is possible to match the dynamic responses of sunspots by news shocks rather well. The solid lines on the right panel of Figure 2 illustrate this property. For these responses, the second coefficient in the matrix M is set to -2.6.

4. Concluding Remarks

This paper compared news and sunspot shocks, as sources of exogenous changes in beliefs, by examining equilibria of linear rational expectations models. It showed that the two types of shocks had different implications for cross-equation restrictions and serial correlation properties of endogenous variables.

The next research question is to ask which type of expectation shocks is more relevant empirically. It is conjectured that the estimation procedure developed by Lubik and Schorfheide (2004) could be adapted to answer this question in a context of a particular model. The relative importance of news and sunspots could be evaluated by comparing a model with fundamental and news shocks in the determinacy region of the parameter space with the same model with fundamental and sunspot shocks in the indeterminacy region of the parameter space. Restricting the possibility of news shocks to the case of determinacy appears justified, given the non-uniqueness of the impact responses to news shocks under indeterminacy and the effect of the news shocks on the endogenous propagation under determinacy.

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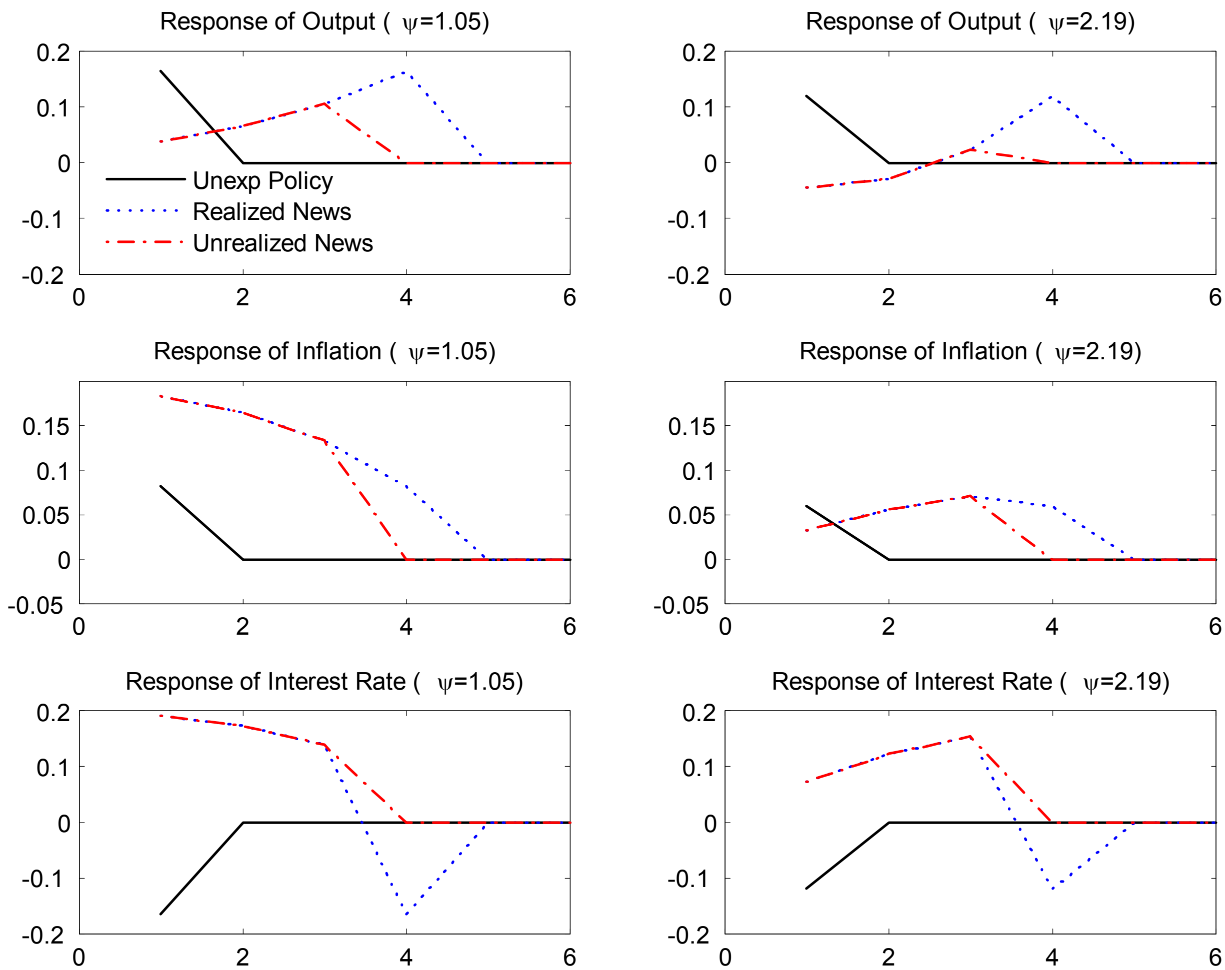
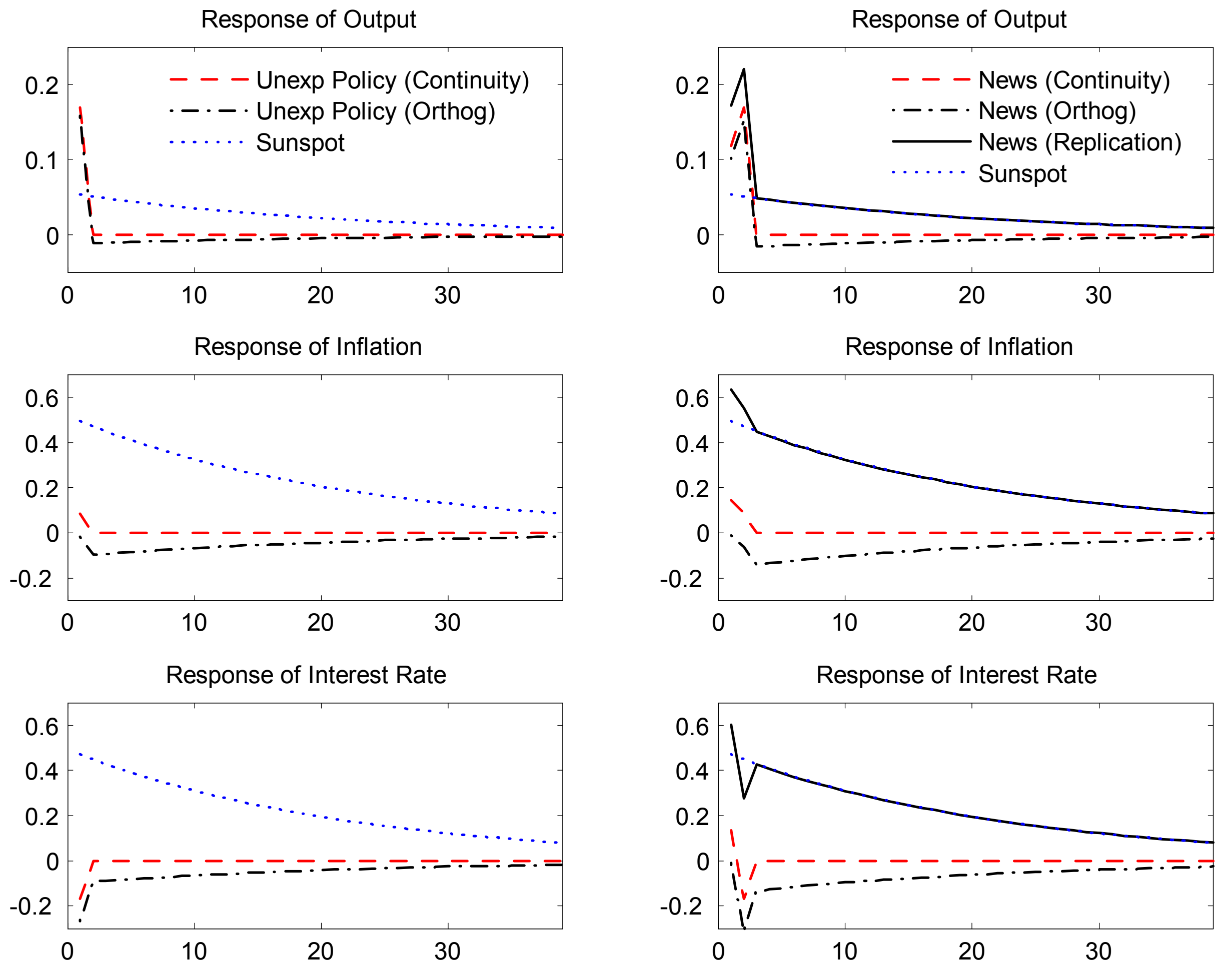
Figure 1. Responses to an unexpected monetary policy expansion, to a realized and unrealized news shocks (Determinacy)



Note: The solid lines characterize the responses of output, inflation and the nominal interest rate to an unexpected 25-basis-point interest rate cut. The other lines represent the responses to a belief, formed in period 1 that in period 4 the interest rate will be cut by 25-basis points. For the dashed lines, this belief is followed by the interest rate cut, as anticipated. For the dashed-dotted lines, the belief is followed by no change in policy. All variables are in percent deviations from the non-stochastic steady state.

Figure 2. Responses to an unexpected monetary policy expansion, to a realized news and a sunspot shocks (Indeterminacy, $\psi = 0.95$)



Note: The dotted lines depict the responses to a reduced form sunspot shock of 0.5%. The dashed and dotted lines correspond to the responses under the 'continuity' and 'orthogonality' assumptions on the rational expectation forecast errors. The left panel contains the responses to an unexpected 25-basis-point interest rate cut. The right panel gives the responses to a belief, formed in period 1 that in period 2 the interest rate will be cut by 25-basis points. This belief is followed by the interest rate cut, as anticipated. The solid lines give the responses to the realized news shock under the 'replication' assumption on the rational expectation forecast errors. All variables are in percent deviations from the non-stochastic steady state.