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Between-Group Transfers and Poverty-Reducing Tax Reforms[#]

by

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Abstract

In this paper, we propose the conception of Within-group Consumption-Dominance Curves in order to capture the impact of indirect tax reforms on poverty. Considering that the population is partitioned into many groups, which differ in needs, in health, in capability or other attributes, we introduce within-group transfers and between-group transfers via indirect taxation mechanisms in order to reduce the poverty for the entire population and the poverty in particular population sub-groups. We show that these taxation schemes are useful for many reasons: there is no need to estimate demand functions; the tax reforms are robust over a large range of poverty indices; the methodology relies on a stochastic dominance approach.

Keywords: *Between-group redistribution; Poverty; Restricted stochastic dominance; Tax reforms.*

JEL classification: *D63, H20*

Résumé

Dans cet article, nous proposons le concept de Courbes de Dominance de Consommation Intra-groupes afin d'analyser l'impact de réformes de la fiscalité indirecte sur la pauvreté. Nous considérons que la population est divisée en plusieurs groupes différant selon les besoins, l'état de santé, les capacités ou quelqu'autre attribut. Nous introduisons des transferts intra-groupes et inter-groupes via le mécanisme de taxation indirecte afin de réduire la pauvreté de la population dans son ensemble et la pauvreté de certains groupes en particulier. Nous montrons que ce cadre d'analyse de taxation est pratique pour plusieurs raisons. Puisque la méthodologie repose sur la dominance stochastique, il n'est pas nécessaire d'estimer les fonctions de demande et les réformes sont robustes pour un large éventail d'indices de pauvreté.

Mots clés: *Redistribution inter-groupe; Pauvreté; Dominance stochastique restreinte; Réformes fiscales.*

Classification JEL: *D63, H20*

1 Introduction

Yitzhaki and Slemrod (1991), subsequently Yitzhaki and Thirsk (1990)¹, Makdissi and Wodon (2002), Liberati (2003), Duclos, Makdissi and Wodon (2005a, 2008), Santoro (2007), Makdissi and Mussard (2008a, 2008b), among others, have introduced and analyzed the impact of transfers between individuals according to indirect taxation frameworks in order to yield decision makers the ability to constitute poverty-reducing or welfare-improving fiscal reforms. These standard tax reforms, for couple of goods $\{i, j\}$, consist in financing a decreasing tax on good i by increasing the tax on good j in a balanced budget framework.

Precisely, Makdissi and Wodon (2002) have initiated the use of a new concept, that of Consumption Dominance curve (CD -curve from now on), which yields the proportion of total consumption of a particular good consumed by the individuals whose income is less than a given income y .² CD -curves enable the impact of marginal indirect tax reforms on poverty to be captured, for any order of restricted stochastic dominance. In other words, if the CD -curve of good i dominates (lies above) that of good j , for any given order, and for all incomes below a defined poverty line, and if the decision maker increases the tax on the j -th commodity and uses the proceeds to subsidize the i -th commodity, then overall poverty declines, and conversely.³

These taxation procedures have many advantages. First, they are compatible with various poverty indices that belong to the set of additive poverty measures. Second, these tax reforms may be implemented for a population of heterogeneous agents, which may differ in needs, in capability, and in other attributes. Third, these tax programs are robust over a large range of poverty lines. Finally, these tests are appealing since they simply relies on a dominance (graphical) approach. They are less restrictive than parametric tests and can be used for all units of consumption expenditures and for any order of restricted stochastic dominance, which corresponds to ethical transfer principles (see Section 2 *infra*).

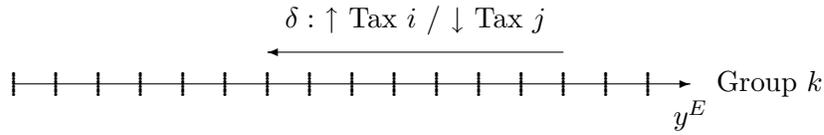
In this paper, partitioning the population in many groups (e.g. gender, health, age, ethnic origin, etc.), we analyze the possibility to make transfers within the groups of the population and between the groups of the population, both concepts being relatively new in this literature. Indeed, instead of taking money directly from higher-income persons to achieve a redistribution among the lower-income ones, we use an indirect-tax mechanism as in Makdissi and Wodon (2002). This yields very intuitive applications. For instance, a within-group transfer (see Figure 1) may be performed by increasing tax on fuel in an urban area in order to subsidize public transportation in the same area, implying poverty reduction.

¹Yitzhaki and Thirsk's (1990) paper was made after, but it was published before.

²This definition corresponds to CD -curves of order 2. See Section 3 *infra* for the other orders.

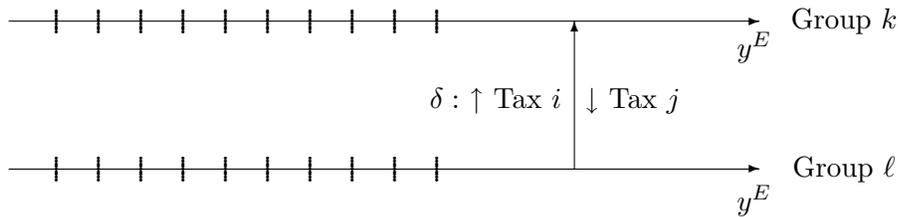
³Duclos, Makdissi and Wodon (2005b) have applied this framework to direct transfer reforms and Makdissi and Wodon (2007) to regulatory reforms.

Figure 1. Within-group Transfer



On the other hand, between-group transfers (see Figure 2) are carried out by taxing the j -th commodity in one group in order to subsidize, in another group, the tax on the same j -th good or alternatively on another commodity. One must think about cross-subsidies between different groups of consumers of a public utility.

Figure 2. Between-group transfer



Within-group and between-group transfers are theoretical developments particularly useful for policy purposes since they can rely on a variety of tax combinations aiming at obtaining two principal results: (i) to decrease poverty in the groups where the tax programs take place (obviously, not in rich groups but principally in the needier groups) and (ii) to decrease poverty for the entire population. This is precisely the aim of our paper, that is, to make use of within- and between-group transfers *via* indirect tax reforms in order to reduce poverty within-groups *and* overall poverty. For this purpose, we propose many tests based on restricted stochastic dominance between within-group *CD*-curves. By definition, *CD*-curves gauge the proportion of total consumption of a particular good in a given group consumed by the individuals whose income is less than a given income y . This allows poverty-reducing tax reforms to be achieved, provided that within-group *CD*-curves do not intersect, for any given order of restricted stochastic dominance. This leads to a set of results for which it is not necessary to impose a same taxation scheme on goods i and j within each group. Indeed, we show that the tax reform may be performed with only two groups of the population, the other tax rates being constant in the other groups. Furthermore, there is the possibility to choose the degree of restricted stochastic dominance: the higher the degree is, the more adverse to poverty the indices are.

The remainder of the paper is organized as follows. Section 2 deals with the assumptions characterizing the analytical forms of the poverty indices and those of the taxation

mechanisms. Section 3 is devoted to the test specification of within- and between-group transfers allowing for poverty-reducing tax reforms to be contemplated. Afterwards, introducing an assumption of fairness, we strengthen the restricted stochastic dominance tests for between-group redistribution and poverty reduction, for any order of dominance. Section 4 yields concluding remarks and advances further researches.

2 Assumptions on poverty and tax reforms

We suggest, on the one hand, a set of assumptions in order to formalize the environment on which we intend to derive our results of restricted stochastic dominance. Let $y^E(q, y)$ represent the *per capita* equivalent income. The notation q symbolizes a vector of unitary market prices e subject to taxes t . We assume that producer's prices always remain constant. Let $\tilde{y}_k(q, y, k)$ represent the *per capita* equivalent income adjusted for differences in demographic characteristics. Those differences may be linked to handicap, gender, ethnicity, age, etc. Usually, $\tilde{y}(q, y, k)$ is linked to $y^E(q, y)$ by an equivalent scale $m(k)$ such that:

$$\tilde{y}(q, y, k) = \frac{y^E(q, y)}{m(k)}. \quad (1)$$

We test for poverty-reducing tax reforms using additive poverty indices. An additive poverty index is defined as the sum of individuals' poverty $p(\cdot)$:

$$P(F, z) = \int_0^a p(\tilde{y}(q, y, k), z) dF(y, k), \quad (2)$$

where $F(y, k)$ is the joint cumulative distribution function of y and k defined over $[0, a]$, a an integer greater than all \tilde{y} , and z the poverty line defined in the adjusted equivalent income space.

As the overall poverty is the sum of individuals' poverty, each agent's equivalent income is compared with a sole common poverty line $z \in \mathbb{R}_{++}$. If the individual's equivalent income is higher or equal to the poverty line z , then $p(y^E(q, y), z) = 0$. Individual's poverty is assumed to be a non-negative function.

When the population of size n is partitioned into K groups of size n_k , $k \in \{1, \dots, K\}$, the advantage of working with such class of additive poverty indices is the possibility to formalize the overall poverty index as the sum of poverty within each population subgroup.

$$P(F, z) = \sum_{k=1}^K \theta_k \int_0^a p(\tilde{y}(q, y, k), z) dF_k(y), \quad (3)$$

where $\theta_k := \frac{n_k}{n}$ is group k 's population share and $F_k(y)$ his cumulative distribution function defined over $[0, a]$. $F_k(y)$ is the proportion of individuals belonging to group k and living in a household with household income lower than or equal to y .

In the literature, there is no consensus on what exactly the analytic form of an equivalent scale $m(k)$ may be. In this context, we can rewrite (3) in a more general form in order to make our approach compatible with various equivalent scales and allowing for analyzing the impact of different equivalent scales on poverty reductions. Let $p_k(y^E(q, y), z_k)$ be the poverty function characterizing group k 's poverty, where z_k is the group specific poverty line defined in the equivalent income space. Let $z_k^+ \in \mathbb{R}_{++}$ being the maximum conceivable poverty line in group k such that $z_k \leq z_k^+$. The function $p_k(y^E(q, y), z_k)$ is non-negative. If an individual of group k has its equivalent income above or equal to the poverty line z_k , then $p_k(y^E(q, y), z_k) = 0$. In this context, the poverty index may be rewritten as a weighted average of poverty intensities within each group:

$$P(F, z) = \sum_{k=1}^K \theta_k \int_0^a p_k(y^E(q, y), z_k) dF_k(y). \quad (4)$$

In the sequel, focus is principally put on poverty indices conceived as a s -time differentiable continuous function almost everywhere over $[0, a]$ such as:

$$(-1)^u p_1^u(\cdot) \geq (-1)^u p_2^u(\cdot) \geq \dots \geq (-1)^u p_k^u(\cdot) \geq 0, \forall u \in \{1, 2, \dots, s\}, \quad (5)$$

where $p_k^u(\cdot)$ is the u -th derivative of the $p_k(\cdot)$ function.

The class of poverty indices satisfying assumptions (4) and (5) is denoted by Π^s . It is a well-suited class of indices (see Duclos and Makdissi (2005), Duclos, Makdissi and Wodon (2005a) and Zheng (1999)) that involves the well-known FGT's measures (Foster, Greer and Thorbecke (1984)). For $s \geq 1$, equation (5) implies that an increase in household equivalent income y^E diminishes poverty, for any given household type. Furthermore, it postulates that for any given household equivalent income y^E , the needier the households are, the greater the poverty alleviation may be. Although the difference in household needs is usually interpreted as gaps in household sizes, this interpretation is less appropriate in our framework. Here, suitable interpretations may be differences in health (handicapped versus non handicapped individuals), gender (women versus men), ethnic and religious affiliations, as well as differences in regions. For any given example, if divergences in capabilities are recorded at the same income level (see Sen (1992)), such an assumption is relevant. Equation (5)'s normative implications are more stringent than the usual ones for first-order unidimensional restricted dominance and can be viewed as a weak version of the Pigou-Dalton principle, which is in fact equivalent to Sen's Weak Equity Axiom (see Sen (1997), p. 18). For $s \geq 2$, equation (5) postulates that an equalizing transfer of $\delta > 0$ from a richer person to a poorer one decreases poverty, this effect being stronger across needier households. Indeed, for higher s , the interpretation of equation (5) can be made using Fishburn and Willig's (1984) general transfer principle, for which increasing weights are associated with transfers occurring at the bottom of the distribution as far as s

increases. Hence, equation (5) makes these properties, viewed as a generalization of Sen's Weak Equity Axiom, normatively more important for needier households. Finally, for any order s , we have Fishburn and Willig's normative interpretation of s -order unidimensional restricted dominance (that is, the interpretation of $(-1)^s p_k^s(\cdot) \geq 0$), coupled with a weak version of the traditional normative interpretation of $(s+1)$ -order restricted dominance (the interpretation of $(-1)^s p_k^s(\cdot) \geq (-1)^s p_{k+1}^s(\cdot)$ in a sequential context).

Now, in order to define the impact of indirect tax reforms on poverty variations, we must consider, on the one hand, the implications of tax reforms on public budget and, on the other hand, these implications on individual wellbeing. Suppose the decision maker intends to marginally increase the tax on commodity j and uses the proceeds to reduce the tax on the i -th commodity. Let M be the number of commodities, $m \in \{1, \dots, M\}$. If $X_m = \int_0^a x_m(y) dF(y)$ denotes the aggregate average consumption of the m -th good, and t_m the tax rate of the m -th good, then the *per-capita* government indirect tax revenue is $R = \sum_{m=1}^M t_m X_m$. Hence, imposing revenue neutrality ($dR = 0$) leads to:

$$\gamma_{ij} = -\frac{dt_j}{dt_i} \left(\frac{X_j}{X_i} \right) = \frac{1 + \frac{1}{X_i} \sum_{m=1}^M t_m \frac{\partial X_m}{\partial t_i}}{1 + \frac{1}{X_j} \sum_{m=1}^M t_m \frac{\partial X_m}{\partial t_j}}, \quad \forall i, j \in \{1, \dots, M\}. \quad (6)$$

Following Wildasin (1984), we may interpret γ_{ij} as the differential economic efficiency cost of raising \$1 of public funds by taxing the j -th commodity *versus* the i -th commodity.

Let us consider the impact of marginal tax reforms on individual wellbeing. Besley and Kanbur (1988) determine the variation of the equivalent income with respect to the tax rate variation of good i . Using Roy's identity and assuming that the observed price vector is the vector of reference, they show that the change in equivalent incomes generated by a marginal change of the tax rate of good i is:

$$\frac{\partial y^E}{\partial t_i} = -x_i(q, y), \quad (7)$$

where $x_i(q, y)$ is the Marshallian demand of good i . On this basis, it is possible to expose our main results.

3 Restricted Stochastic Dominance and Poverty-Reducing Tax Reforms

Building on the framework described in the previous section, Makdissi and Wodon (2002) define CD -curves in order to perform s -order restricted stochastic dominance tests. The CD -Curve of order 1 for good i is the ratio between an individual consumption with income y and the aggregate consumption of good i : $CD_i^1(y) = x_i(y)/X_i \cdot f(y)$, where $f(y)$ is the density function of *per capita* income, which is nil outside of the interval $[0, a]$.⁴ The CD -

⁴Makdissi and Wodon (2002) use the following definition $CD_i^1(y) = x_i(y)/X_i$. However, Duclos, Makdissi and Wodon (2008) show that it is more helpful to use $CD_i^1(y) = x_i(y)/X_i \cdot f(y)$ for estimation purposes.

curve of order s is given by: $CD_i^s(y) = \int_0^y CD_i^{s-1}(u)du$. For instance, the CD -curve of order 2 of good i represents the share of total consumption of good i consumed by the individuals whose income is less than y . Note that the CD -curves are obtained by integrating the consumption expenditures over the income space. Integrating over the percentile space would provide the traditional concentration curves.⁵

Using those CD -curves, Makdissi and Wodon (2002) derive restricted dominance conditions for poverty-reducing indirect tax reforms. To tackle the problem of differences in needs between households, Duclos, Makdissi and Wodon (2005a) use equation (4) and define the CD -Curve of order 1 for good i and for group k . It represents the ratio between an individual consumption of group k with income y and the aggregate consumption of good i : $\widetilde{CD}_{ik}^1(y) = x_{ik}(y)/X_i \cdot f_k(y)$, where $f_k(y)$ is the density function of *per capita* incomes of group k , with $f_k(y) = 0$ outside of the interval $[0, a]$. Subsequently, the s -order CD -curve of group k for good i is given by: $\widetilde{CD}_{ik}^s(y) = \int_0^y \widetilde{CD}_{ik}^{s-1}(u)du$.

In this respect, Duclos, Makdissi and Wodon (2005a) provides a sufficient condition to obtain a poverty-reducing tax reform with heterogeneous agents. Increasing the tax on the j -th commodity and decreasing the tax on the i -th commodity, that is $dt_j = -\gamma_{ij} \frac{X_i}{X_j} dt_i > 0$, implies a poverty reduction for the entire population. Indeed, it is then sufficient that the sum of the \widetilde{CD} -curves of good i lies above that of good j (provided that the \widetilde{CD} curves of good j are multiplied by γ_{ij}). This result appears in the following theorem, for which we provide the necessary condition.

Theorem 3.1 *A revenue-neutral marginal tax reform, $dt_j = -\gamma_{ij} \frac{X_i}{X_j} dt_i > 0$, $dt_m = 0 \forall m \neq i, j \in \{1, 2, \dots, M\}$, will reduce poverty $\forall P(F, z) \in \Pi^s$, $s \in \{1, 2, 3, \dots\}$ and $\forall z_k \leq z_k^+$, $\forall k \in \{1, 2, \dots, K\}$, if and only if*

$$\sum_{k=1}^{\ell} \theta_k [\widetilde{CD}_{ik}^s(y) - \gamma_{ij} \widetilde{CD}_{jk}^s(y)] \geq 0, \forall y \leq z_{\ell}^+, \forall \ell \in \{1, 2, \dots, K\}.$$

Proof. See the appendix. ■

Theorem 3.1 shows that dominance between \widetilde{CD} -curves within each group is not necessary to reduce poverty since the *iff* condition holds for the dominance of the sum. For instance, suppose we decrease the tax on good i (alcoholic drinks) and we increase the tax on good j (tobacco). If the \widetilde{CD} -curve of alcoholic drinks in the most needy group (group 1) dominates that of tobacco (multiplied by γ_{ij}), and if this dominance is strong enough to compensate for the non dominance within the other groups, then the tax reform decreases the overall poverty.

Theorem 3.1 is very useful for many reasons. First, there is no need to estimate demand functions, since the observed demand of the goods (given by the envelope theorem, see

⁵For an analysis on how concentration curves may be used to derive conditions for each order of positional dominance the reader may refer to Makdissi and Mussard (2008a, 2008b).

equation (7)) are given by the surveys of household spending. Second, the tax reforms are robust over a large number of poverty indices included in Π^s . Finally, although the heterogeneity of the agents' needs entails many analytical complications, the methodology is quite simple since it relies on a dominance approach.

In spite of their attractiveness, the implications of Theorem 3.1 are only concerned with overall poverty. Indeed, it is implicitly assumed in Theorem 3.1 that tax rates t_i and t_j are homogeneous over all demographic groups. Consequently, decreasing overall poverty with a marginal tax reform, such that $dt_j = -\gamma_{ij} \frac{X_i}{X_j} dt_i > 0$, entails in some cases a reduction of poverty for which particular groups (maybe the needier one) can be more solicited than others. Consequently, in order to avoid this criticism, we introduce an axiom of equal treatment of the groups.

Axiom 3.1 : *Equal Treatment of the Groups.* Let R^k be the fiscal revenue obtained in group k . Revenue neutrality is the rule used in each k group, if and only if:

$$dR^k = 0, \forall k \in \{1, \dots, K\}. \quad (8)$$

The equal treatment of the groups may be coherent with a decentralized poverty alleviation program in which each region must have a balanced budget. Subsequently, we include an assumption to improve fairness in the indirect taxation scheme. Indeed, we introduce the possibility to investigate the case of a *per-group* taxation design, that is, each group of the population imposes his own tax rates, $t_i^k, t_j^k, \forall k \in \{1, \dots, K\}$ on any couple of goods $\{i, j\} \in \{1, \dots, M\}$.

In such a context, instead of examining \widetilde{CD} -curves of any commodity of group k based on the aggregate consumption of the population, we investigate for CD -curves associated with the aggregate consumption of each group $k \in \{1, \dots, K\}$. The first-order within-group CD -curve of group k for good i is the ratio between an individual consumption with income y and the aggregate consumption of his group k for good i : $CD_{ik}^1(y) = x_{ik}(y)/X_{ik} \cdot f_k(y)$. Thus, the s -order within-group CD -curve of group k for good i is given by: $CD_{ik}^s(y) = \int_0^y CD_{ik}^{s-1}(u) du$.

Then, using the *per-group* taxation scheme together with Axiom 3.1, we assume the decision maker marginally increases the tax on commodity j for all groups and uses the proceeds in each group to reduce the tax on commodity i . Budget neutrality within groups yields the following within-group differential efficiency parameter:

$$\tilde{\gamma}_{ij}^k = -\frac{dt_j^k}{dt_i^k} \left(\frac{X_{jk}}{X_{ik}} \right) = \frac{1 + \frac{1}{X_{ik}} \sum_{m=1}^M t_m^k \frac{\partial X_{mk}}{\partial t_i^k}}{1 + \frac{1}{X_{jk}} \sum_{m=1}^M t_m^k \frac{\partial X_{mk}}{\partial t_j^k}}. \quad (9)$$

Now, by invoking the assumptions of additive poverty (4) and differentiability (5), it is possible to state a theorem with the condition of equal treatment of the groups. This

yields an analogous result as Theorem 3.1, except we impose the revenue-neutral condition group by group in order to deal with the heterogeneity of the demand curves.

Theorem 3.2 *Under Axiom 3.1, a revenue-neutral marginal tax reform, $dt_j^k = -\tilde{\gamma}_{ij}^k \frac{X_{ik}}{X_{jk}} dt_i^k > 0 \forall k \in \{1, 2, \dots, K\}$ with $dt_m^k = 0 \forall m \neq i, j \in \{1, 2, \dots, M\}$, will reduce poverty $\forall P(F, z) \in \Pi^s$ $s \in \{1, 2, 3, \dots\}$ and $\forall z_k \leq z_k^+$, $\forall k \in \{1, 2, \dots, K\}$, if and only if*

$$\sum_{k=1}^{\ell} [CD_{ik}^s(y) - \tilde{\gamma}_{ij}^k CD_{jk}^s(y)] \geq 0, \quad \forall y \leq z_{\ell}^+, \quad \forall \ell \in \{1, 2, \dots, K\}.$$

Proof. See the appendix. ■

As in Theorem 3.1, the result is attractive since overall poverty decreases if, and only if, there is dominance of the sum of the within-group CD -curves for good i (provided that those of good j are multiplied by $\tilde{\gamma}_{ij}^k$). The condition is that we increase t_j^k for all k and use the proceeds to subsidize t_i^k for all k . The advantage of this result relies on the fact that the taxation scheme provides consequential freedom to the decision maker with respect to the groups she decides to subsidize. Indeed, poverty-reducing tax reforms may be performed by taxing one or many groups without affecting the remainder of the population. For instance, the tax reform may be concerned with group k only (see Corollary 3.1 *infra*).

Corollary 3.1 *Under Axiom 3.1, a revenue-neutral marginal tax reform, $dt_j^k = -\tilde{\gamma}_{ij}^k \frac{X_{ik}}{X_{jk}} dt_i^k > 0$ with $dt_m^k = 0 \forall m \neq i, j \in \{1, 2, \dots, M\}$ and $dt_m^{\ell} = 0 \forall \ell \neq k \in \{1, 2, \dots, K\}$ and $\forall m \in \{1, 2, \dots, M\}$, will reduce poverty $\forall P(F, z) \in \Pi^s$, $s \in \{1, 2, 3, \dots\}$ and $\forall z_k \leq z_k^+$, if and only if*

$$CD_{ik}^s(y) - \tilde{\gamma}_{ij}^k CD_{jk}^s(y) \geq 0, \quad \forall y \leq z_k^+.$$

Proof. It is straightforward. ■

In other words, if a tax reform only occurs in group k (tax-rate variations being nil in the other groups), then the overall poverty declines. For instance, suppose the population is partitioned into two groups: urban area (k) and rural area (ℓ). Hence, it is possible to finance an increasing subsidy on public transport by an increasing tax on fuel in group k . This yields incentives with overall poverty reduction provided the within-group CD -curve of public transport lies nowhere below that of fuel in this area (and provided the latter is multiplied by $\tilde{\gamma}_{ij}^k$), for any chosen order of restricted stochastic dominance. This is typically the result of a *within-group transfer* (see Figure 1 *supra*).

Now, imagine a tax reform is performed in group k where the number of poor individuals is important. This may lead to some problems if the proceeds issued from the fiscal revenue are low. Then, instead of increasing the tax on the j -th commodity to subsidize the i -th commodity in the same group, why not financing a decreasing tax in a poor group with an increasing tax in a rich group? In such a taxation environment, implying *between-group*

transfers, both fiscal revenue and fairness may be improved. For this purpose, we invoke a fair-treatment-of-the-group axiom.

Axiom 3.2 : Fair Treatment of the Groups. Let $R_{ij}^{k\ell}$ be the per capita indirect tax revenue obtained from group k and ℓ . Revenue neutrality is assumed to be the rule between groups k and ℓ , if we finance a decreasing tax on good i in group k by an increasing tax on good j in group ℓ :

$$dR_{ij}^{k\ell} = 0, \text{ for any } k, \ell \in \{1, \dots, K\} \text{ and for any } i, j \in \{1, \dots, M\}. \quad (10)$$

The fair treatment of the groups may yield relevant between-group transfers with a matter of progressiveness into the indirect tax mechanism with respect to the *per-group* taxation scheme. Given Axiom 3.2, we obtain a between-group differential efficiency parameter:

$$\widehat{\gamma}_{ij}^{\ell k} = -\frac{dt_j^\ell}{dt_i^k} \left(\frac{X_{j\ell}}{X_{ik}} \right) = \frac{\theta_\ell \left[1 + \frac{1}{X_{j\ell}} \sum_{m=1}^M t_m^\ell \frac{\partial X_{m\ell}}{\partial t_j^\ell} \right]}{\theta_k \left[1 + \frac{1}{X_{ik}} \sum_{m=1}^M t_m^k \frac{\partial X_{mk}}{\partial t_i^k} \right]}. \quad (11)$$

Note that, in contrast to the within-group differential efficiency ratio, the between-group one includes population shares of the groups concerned with the tax reform. Remember that a (within-group) differential efficiency ratio gauges, under budget neutrality condition, the *per-capita* budgetary impact of the reform (in each group). Consequently, in a two-group taxation framework, using the weights of population shares enables us to assess the total impact on the public budget. Indeed, imagine we finance a decreasing tax on the i -th commodity in one group (say k) with an increasing tax on the j -th commodity in another group (say ℓ). If $\theta_\ell \gg \theta_k$, the decreasing tax on good i in group k can be performed with a very marginal growth of t_j^ℓ . Accordingly, it is possible to perform between-group transfers *via* poverty-reducing tax reforms (see the following Theorem and Figure 2 *supra*).

Theorem 3.3 Under Axiom 3.2, a revenue-neutral marginal tax reform, $dt_j^\ell = -\widehat{\gamma}_{ij}^{\ell k} \frac{X_{ik}}{X_{j\ell}} dt_i^k > 0$ with $dt_m^\ell = 0 \forall m \neq j \in \{1, 2, \dots, M\}$, $dt_m^k = 0 \forall m \neq i \in \{1, 2, \dots, M\}$, $dt_m^h = 0 \forall m \in \{1, 2, \dots, M\} \forall h \neq \ell, k \in \{1, 2, \dots, K\}$, will reduce poverty $\forall P(F, z) \in \Pi^s$, $s \in \{1, 2, 3, \dots\}$ and $\forall z_h \leq z_h^+$ with $h \in \{k, \ell\}$, if and only if

$$CD_{ik}^s(y) - \widehat{\gamma}_{ij}^{\ell k} CD_{j\ell}^s(y) \geq 0, \forall y \leq z_\ell^+.$$

Proof. See the appendix. ■

In Theorem 3.3, we do not have to test for the neediest group's within-group CD curve. Indeed, this group necessarily gains from the reform since his tax rate decreases and the cost of this tax reduction is supported by another group. This result is very helpful to get between-group transfers by taxing a rich group and to subsidize a needier group with a two-good taxation scheme. For instance, we know that Canadian natives are

exempted from VAT. Then, assume j represents all goods, whereas i stands for housing expenditures. Consequently, a policy aiming at increasing subsidizes on housing for natives may be achieved with a slight increasing VAT for which non natives are solicited only.

Alternatively, this technique may be applied in the one-good case, which is useful when we consider cross-price subsidies for public utilities between different consumer groups.

Corollary 3.2 . *Under Axiom 3.2, a revenue-neutral marginal tax reform, $dt_i^\ell = -\hat{\gamma}_{ii}^{\ell k} \frac{X_{ik}}{X_{i\ell}} dt_i^k > 0$ with $dt_m^\ell = 0 \forall m \neq i \in \{1, 2, \dots, M\}$, $dt_m^k = 0 \forall m \neq i \in \{1, 2, \dots, M\}$, $dt_m^h = 0 \forall m \in \{1, 2, \dots, M\} \forall h \neq \ell, k \in \{1, 2, \dots, K\}$, will reduce poverty $\forall P(F, z) \in \Pi^s$, $s \in \{1, 2, 3, \dots\}$ and $\forall z_h \leq z_h^+$ with $h \in \{k, \ell\}$, if and only if*

$$CD_{ik}^s(y) - \hat{\gamma}_{ii}^{\ell k} CD_{i\ell}^s(y) \geq 0, \forall y \leq z_\ell^+.$$

Proof. See the appendix. ■

Finally, contrary to the previous results, Theorem 3.3 and Corollary 3.2 allow to test for restricted stochastic dominance between two within-group curves associated with two distinct population sub-groups. Therefore, a *per-group* taxation model is relevant with between-group transfers, provided that commodity consumptions in a given group are more concentrated among the poor than in the other group.

4 Conclusion

Indirect tax reforms exert, throughout the use of within-group *CD*-curves, a non-homogenous impact on the distributions of commodity expenditures since the underlying *per-group* taxation assumption yields modifications of consumption habits group by group. This confers decision makers the ability to perform within-group transfers as well as between-group transfers to reduce poverty in particular groups *and* to obtain an overall poverty alleviation. In poverty measurement, within- and between-group transfers are usually made directly from rich to poor households. In this paper, we lay the emphasize on the implementation of within- and between-group transfers and we imagine a *per-group* taxation scheme in order to introduce some “progressiveness” into the indirect taxation system aiming at obtaining poverty reductions.

The technique possesses many advantages. It is relatively simple since applications rely on graphical approaches. Furthermore, there is no need to estimate demand functions, and the poverty-reducing tax reforms comprise a large variety of poverty indices and also a large range of poverty lines.

This methodology may contribute to open the way on new topics. Indeed, between-group indirect tax reforms may be studied to capture the impact on the reduction of inequalities, of between-group inequalities using the Gini index between populations of income receivers (see Dagum (1987)), or to analyze mobility variations (see e.g. Van Kerm

(2004)). Moreover, it would be interesting to adapt these restricted stochastic dominance tests in order to apprehend the dynamics of reducing-poverty tax reforms as well as the efficiency of the redistribution mechanism by measuring their statistical significance (e.g. with Davidson and Duclos's (2000) test).

Finally, this paper opens the way on another crucial point, which is the analysis of poverty-reducing indirect tax reforms with multiple goods including the nature of the goods (luxury goods or inferior goods). This is left for future researches.

Appendix

For the demonstrations of the theorems, one needs Abel's lemma.

Lemma 4.1 : Abel's lemma (see Jenkins and Lambert (1993)). *Let x_j and y_i be two real variables. If $x_n \geq x_{n-1} \geq \dots \geq x_1 \geq 0$, then $\sum_{i=j}^n y_i \geq 0 \forall j$ is a sufficient condition for $\sum_{i=1}^n x_i y_i \geq 0$. Contrary to this, if $x_n \leq x_{n-1} \leq \dots \leq x_1 \leq 0$, then $\sum_{i=j}^n y_i \geq 0 \forall j$ is also a sufficient condition for $\sum_{i=1}^n x_i y_i \leq 0$.*

Proof. Theorem 3.1.

(Sufficiency): Sufficiency is due to Duclos, Makdissi and Wodon (2005a). For simplicity, let $p_k^{(s)}(\cdot) := p_k^{(s)}(y^E(q, y), z_k)$. Following Eq. (7), the impact of the indirect tax reform on group k 's poverty is:

$$dp_k(\cdot) = p_k^{(1)}(\cdot)x_{ik}dt_i + p_k^{(1)}(\cdot)x_{jk}dt_j.$$

Following the indirect tax reform, $dt_j = -\gamma_{ij}\frac{X_i}{X_j}dt_i > 0$, and Eq. (6), we have:

$$dp_k(\cdot) = -p_k^{(1)}(\cdot) \left[\frac{x_{ik}}{X_i} - \gamma_{ij} \frac{x_{jk}}{X_j} \right] X_i dt_i.$$

This expression may be rewritten with \widetilde{CD} -curves:

$$dp_k(\cdot) = -p_k^{(1)}(\cdot) \left[\widetilde{CD}_{ik}^1(y) - \gamma_{ij} \widetilde{CD}_{jk}^1(y) \right] X_i dt_i.$$

Since the aggregate poverty (4) is:

$$P(F, z) = \sum_{k=1}^K \theta_k \int_0^a p_k(y^E(q, y), z_k) dF_k(y),$$

the impact of the tax reform on the aggregate poverty is:

$$dP = -X_i dt_i \sum_{k=1}^K \theta_k \int_0^a p_k^{(1)}(\cdot) \left[\widetilde{CD}_{ik}^1(y) - \gamma_{ij} \widetilde{CD}_{jk}^1(y) \right] dy.$$

Integrating by parts s times $\int_0^a p_k^{(1)}(\cdot) \widetilde{CD}_{ik}^1(y) dy$ and using an induction reasoning yields:

$$\int_0^a p_k^{(1)}(\cdot) \widetilde{CD}_{ik}^1(y) dy = (-1)^{s-1} \int_0^a p_k^{(s)}(\cdot) \widetilde{CD}_{ik}^s(y) dy.$$

This entails:

$$dP = (-1)^s X_i dt_i \sum_{k=1}^K \int_0^a p_k^{(s)}(\cdot) \theta_k \left[\widetilde{CD}_{ik}^s(y) - \gamma_{ij} \widetilde{CD}_{jk}^s(y) \right] dy.$$

Using Abel's lemma, if $\sum_{k=1}^{\ell} \theta_k \left[\widetilde{CD}_{ik}^s(y) - \gamma_{ij} \widetilde{CD}_{jk}^s(y) \right] \geq 0 \forall y \leq z_\ell$ and $\forall \ell \in \{1, 2, \dots, K\}$, then $dP \leq 0$.

(*Necessity*): Let us take a set of functions $p_k(y^E(q, y), z_k)$, for which the $(s-1)$ -th derivative is:

$$p_k^{(s-1)}(y^E(q, y), z_k) = \begin{cases} (-1)^{s-1} \epsilon & \text{if } y \leq \bar{y} \\ (-1)^{s-1} (\bar{y} + \epsilon - y) & \text{if } \bar{y} < y \leq \bar{y} + \epsilon, \\ 0 & y > \bar{y} + \epsilon \\ 0 & \forall k \in \{\ell + 1, \dots, K\}. \end{cases} \quad \forall k \in \{1, 2, \dots, \ell\}$$

Poverty indices whose functions $p_k(y^E(q, y), z_k)$ have the above form for $p_k^{(s-1)}(y^E(q, y), z_k)$ belong to the class Π^s . This yields:

$$p_k^{(s)}(y^E(q, y), z_k) = \begin{cases} 0 & \text{if } y \leq \bar{y} \\ (-1)^s & \text{if } \bar{y} < y \leq \bar{y} + \epsilon, \\ 0 & y > \bar{y} + \epsilon \\ 0 & \forall k \in \{\ell + 1, \dots, K\}. \end{cases} \quad \forall k \in \{1, 2, \dots, \ell\} \quad (12)$$

Imagine now that $\sum_{k=1}^{\ell} \theta_k [\widetilde{CD}_{ik}^s(y) - \gamma_{ij} \widetilde{CD}_{jk}^s(y)] < 0$ on an interval $[\bar{y}, \bar{y} + \epsilon]$ for some ℓ , for $\bar{y} < z_\ell^+$, and for ϵ that can be arbitrarily close to 0. For $p_k(y^E(q, y), z_k)$ indices with s -order derivatives defined as in (12), the marginal tax reform induces an increase of poverty. Hence it cannot be that $\sum_{k=1}^{\ell} \theta_k [\widetilde{CD}_{ik}^s(y) - \gamma_{ij} \widetilde{CD}_{jk}^s(y)] < 0$ for some ℓ , $y \in [\bar{y}, \bar{y} + \epsilon]$ when $\bar{y} < z_\ell^+$. This proves necessity. ■

Proof. Theorem 3.2.

(*Sufficiency*): In this context:

$$dp_k(\cdot) = -p_k^{(1)}(\cdot) \left[\frac{x_{ik}(y)}{X_{ik}} - \tilde{\gamma}_{ij}^k \frac{x_{jk}(y)}{X_{jk}} \right] X_{ik} dt_i^k.$$

Now, remember that $\frac{x_{ik}(y)}{X_{ik}} \cdot f(y) = CD_{ik}^1(y)$, then:

$$dP(F, z) = - \sum_{k=1}^K \underbrace{(dt_i^k \theta_k X_{ik})}_{\lambda_k} \int_0^a p_k^{(1)}(\cdot) \left[CD_{ik}^1(y) - \tilde{\gamma}_{ij}^k CD_{jk}^1(y) \right] dy.$$

Integrating by parts $\int_0^a p_k^{(1)}(\cdot)CD_{ik}^1(y)dy$ and $\int_0^a p_k^{(1)}(\cdot)CD_{jk}^1(y)dy$ s times and using an induction reasoning implies that:

$$\begin{aligned} dP(F, z) &= (-1)^s \sum_{k=1}^K \lambda_k \int_0^a p_k^{(s)}(\cdot) \left[CD_{ik}^s(y) - \tilde{\gamma}_{ij}^k CD_{jk}^s(y) \right] dy \\ &= \int_0^a \sum_{k=1}^K \lambda_k (-1)^s p_k^{(s)}(\cdot) \left[CD_{ik}^s(y) - \tilde{\gamma}_{ij}^k CD_{jk}^s(y) \right] dy. \end{aligned}$$

As $dt_i^k < 0$ for all $k \in \{1, \dots, K\}$, it can be noticed that $\lambda_k < 0$, for all k . Using Abel's lemma, in order to get $\sum_{k=1}^K (-1)^s p_k^{(s)} \lambda_k [CD_{ik}^s(y) - \tilde{\gamma}_{ij}^k CD_{jk}^s(y)] \leq 0$, it is then sufficient to have $\sum_{k=1}^\ell [CD_{ik}^s(y) - \tilde{\gamma}_{ij}^k CD_{jk}^s(y)] \geq 0, \forall \ell \in \{1, 2, \dots, K\}$. Thus, $\sum_{k=1}^\ell [CD_{ik}^s(y) - \tilde{\gamma}_{ij}^k CD_{jk}^s(y)] \geq 0, \forall \ell \in \{1, 2, \dots, K\}, \forall y \in [0, z_\ell^+]$, implies $dP(F, z) \leq 0$.

(Necessity): The proof is similar to the one of Theorem 3.1. ■

Proof. Theorem 3.3 and Corollary 3.2.

(Sufficiency): We only present the sufficiency of Theorem 3.3, that of Corollary 3.2 being a particular case for $j = i$. In this context, we have:

$$dp_k(\cdot) = -p_k^{(1)}(\cdot) \left[\frac{x_{ik}(y)}{X_{ik}} \right] X_{ik} dt_i^k$$

and,

$$dp_\ell(\cdot) = -p_\ell^{(1)}(\cdot) x_{j\ell} dt_j^\ell,$$

since $dt_m^\ell = 0 \forall m \neq j \in \{1, \dots, M\}$, $dt_m^k = 0 \forall m \neq i \in \{1, \dots, M\}$, $dt_m^h = 0 \forall m \in \{1, \dots, M\} \forall h \neq \ell, k \in \{1, 2, \dots, K\}$. Then,

$$dp_\ell(\cdot) = -p_\ell^{(1)}(\cdot) \frac{x_{j\ell}}{X_{j\ell}} \left[-\gamma_{ij}^{\ell k} X_{ik} dt_i^k \right],$$

with,

$$\gamma_{ij}^{\ell k} = -\frac{dt_j^\ell}{dt_i^k} \left(\frac{X_{j\ell}}{X_{ik}} \right) = \frac{\left[1 + \frac{1}{X_{j\ell}} \sum_{m=1}^M t_m^\ell \frac{\partial X_{m\ell}}{\partial t_j^\ell} \right]}{\left[1 + \frac{1}{X_{ik}} \sum_{m=1}^M t_m^k \frac{\partial X_{mk}}{\partial t_i^k} \right]}.$$

This entails:

$$dP(F, z) = -\theta_k \int_0^a p_k^{(1)}(\cdot) CD_{ik}^1(y) X_{ik} dt_{ik} dy - \theta_\ell \int_0^a p_\ell^{(1)}(\cdot) CD_{j\ell}^1(y) \left(-\gamma_{ij}^{\ell k} X_{ik} dt_{ik} \right) dy.$$

After integrating each term by parts s times, we have:

$$dP(F, z) = (-1)^s X_{ik} dt_{ik} \theta_k \int_0^a p_k^{(s)}(\cdot) CD_{ik}^s dy + (-1)^s \frac{\theta_\ell}{\theta_k} \left(-\theta_k \gamma_{ij}^{\ell k} X_{ik} dt_{ik} \right) \int_0^a p_\ell^{(s)}(\cdot) CD_{j\ell}^s(y) dy.$$

Let $\lambda_k = dt_i^k \theta_k X_{ik}$ and remember that,

$$\widehat{\gamma}_{ij}^{\ell k} = -\frac{dt_j^\ell}{dt_i^k} \left(\frac{X_{j\ell}}{X_{ik}} \right) = \frac{\theta_\ell \left[1 + \frac{1}{X_{j\ell}} \sum_{m=1}^M t_m^\ell \frac{\partial X_{m\ell}}{\partial t_j^\ell} \right]}{\theta_k \left[1 + \frac{1}{X_{ik}} \sum_{m=1}^M t_m^k \frac{\partial X_{mk}}{\partial t_i^k} \right]},$$

it then follows that:

$$dP(F, z) = (-1)^s \lambda_k \int_0^a \left[\left(p_k^{(s)}(\cdot) - p_\ell^{(s)}(\cdot) \right) CD_{ik}^s(y) + p_\ell^{(s)}(\cdot) \left(CD_{ik}^s(y) - \widehat{\gamma}_{ij}^{\ell k} CD_{j\ell}^s(y) \right) \right] dy. \quad (13)$$

Remember that $\lambda_k < 0$. As we subsidize group k (needier than ℓ), we have $k < \ell$, then $(-1)^s p_k^{(s)}(\cdot) - (-1)^s p_\ell^{(s)}(\cdot) \geq 0$. Consequently, the sufficient conditions for $dP \leq 0$ $\forall P(F, z) \in \Pi^s$ $s \in \{1, 2, 3, \dots\}$ and $\forall z_h \leq z_h^+$ with $h \in \{k, \ell\}$ are:

$$CD_{ik}^s(y) - \widehat{\gamma}_{ij}^{\ell k} CD_{j\ell}^s(y) \geq 0, \forall y \leq z_\ell^+.$$

(*Necessity*): The proof is similar to the one of Theorem 3.1. Let us take a set of functions $p_h(y^E(q, y), z_h)$, for which the $(s-1)$ -th derivative is:

$$p_h^{(s-1)}(y^E(q, y), z_h) = \begin{cases} (-1)^{s-1} \epsilon & \text{if } y \leq \bar{y} \\ (-1)^{s-1} (\bar{y} + \epsilon - y) & \text{if } \bar{y} < y \leq \bar{y} + \epsilon, \quad \forall h \in \{k, \ell\}, \\ 0 & y > \bar{y} + \epsilon. \end{cases}$$

Poverty indices whose functions $p_h(y^E(q, y), z_h)$ have the above form for $p_h^{(s-1)}(y^E(q, y), z_h)$ belong to the class Π^s . This yields:

$$p_h^{(s)}(y^E(q, y), z_h) = \begin{cases} 0 & \text{if } y \leq \bar{y} \\ (-1)^s & \text{if } \bar{y} < y \leq \bar{y} + \epsilon, \quad \forall h \in \{k, \ell\}, \\ 0 & y > \bar{y} + \epsilon. \end{cases} \quad (14)$$

For this particular form of $p_h(y^E(q, y), z_h)$, $(p_k^{(s)}(\cdot) - p_\ell^{(s)}(\cdot)) CD_{ik}^s(y) = 0$ and equation (13) becomes

$$dP(F, z) = (-1)^s \lambda_k \int_0^a p_\ell^{(s)}(\cdot) \left(CD_{ik}^s(y) - \widehat{\gamma}_{ij}^{\ell k} CD_{j\ell}^s(y) \right) dy.$$

Suppose by contraposition that $CD_{ik}^s(y) - \widehat{\gamma}_{ij}^{\ell k} CD_{j\ell}^s(y) < 0, \forall y \leq z_h^+$ on an interval $[\bar{y}, \bar{y} + \epsilon]$ for $h \in \{k, \ell\}$, for $\bar{y} < z_h^+$, and for ϵ that can be arbitrarily close to 0. For poverty indices $p_h^{(s)}(y^E(q, y), z_h)$ defined as in (14), we obtain $P(F, z) > 0$ which is a contradiction. This proves necessity. ■

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