Are Canadian Regional Business Cycles All Alike?

by

Gabriel Rodriguez

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Gabriel Rodríguez\footnote{Address for Correspondence: Department of Economics, University of Ottawa, P. O. Box 450, Station A, Ottawa, Ontario, Canada, K1N-6N5. E-mail address: gabrielr@uottawa.ca.}
Department of Economics
University of Ottawa

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Abstract

Using three different econometric methodologies, this paper identifies business cycles fluctuations in Canadian regions using quarterly real GDP for the period 1961:1 - 2000:1. With the estimates of the transitory and permanent components, as well as filtered and smoothed probabilities of being in times of recession, a chronology for the regional business cycles fluctuations is presented. For the purpose of comparison, results for Canada and the US are also included. Lastly, evidence about correlations between different business cycles is also presented.

Keywords: Regional Business Fluctuations, Markov Switching, Unobserved Components.

JEL Classification: C2, C3, C5.

Résumé

Utilisant trois différentes méthodologies économétriques, ce travail identifie les fluctuations dans les cycles économiques des régions canadiennes utilisant les données trimestrielles sur le PIB réel pour la période de 1961:1 à 2000:1. Avec les estimés des composantes transitoires et permanentes, ainsi qu’à l’aide des probabilités d’être en période de récession, un ordre chronologique pour les fluctuations des cycles économiques régionaux est établi. Pour des fins de comparaison, les résultats pour le Canada, ainsi que pour les États-Unis sont présentés. Dernièrement, l’évidence de corrélation entre les différents cycles économiques est aussi présentée.

Mots-clés: Fluctuations économiques régionales, changements de régime de Markov, composantes inobservées.

Code JEL: C2, C3, C5.
1 Introduction

According to Burns and Mitchell (1946), “business cycles are a type of fluctuations found in the aggregate economic activity of nations that organize their work mainly in business enterprises; a cycle consists of expansions occurring at about the same time in many economic activities followed by similarly general recessions, contractions, and revivals which merge into the expansion phase of the next cycle” (p.3). This definition possesses two basic features. The first is the co-movement among individual economic variables, taking into account possible leads and lags in timing, including the historical concordance of hundred of series such as commodity output, income, prices, interest rates, banking transactions, and transportation services. The second feature is how business cycles are divided into separate phases or regimes.

Research in the literature on the persistence of shocks has imposed symmetry as is the case, for example of Nelson and Plosser (1982), Campbell and Mankiw (1987), Cochrane (1988) and Watson (1986). Some deviations from this framework such as the Beaudry and Koop (1993) have shown that negative innovations to output are less persistent than positive ones. Cover (1992) also offer evidence about this issue when he discussed money-supply innovations.

Presence of asymmetries implies that contractions in business cycles are on average shorter and steeper than expansions (see, for example, Zarnowitz, 1992; Kim and Nelson, 1999b). Neftci (1984) presented empirical evidence of the kind of asymmetry observed before by Friedman (1964) and Keynes (1936). In fact, Neftci (1984) found that the unemployment rate is characterized by sudden jumps and slower declines. Further evidence about this behavior is found in Delong and Summers (1986), Falk (1986), and Sichel (1993). These types of asymmetries are consistent with the Friedman plucking model and also with models where recessions are occasioned by infrequent permanent negative shocks as in the Markov-Switching models of Hamilton (1989) and Lam (1990). See also Kim and Nelson (1999b) and duration dependence models as in Diebold and Rudebusch (1990), Diebold, Rudebusch and Sichel (1993), and Durland and McCurdy (1994) in an univariate context. In another hand, Kim and Nelson (1998) study this issue in a multivariate context. All these references find empirical support for duration dependence only for recessions. Then, the longer a recession persists, the more likely it is to end.

This paper uses three different simple econometric specifications applied to the logarithm of the real quarterly GDP of Canadian regions covering the period from 1961:1 until 2000:1 with the goal being the identification
of business cycle fluctuations in these territories. In order for comparison, estimations for Canada and the US are also included. To my knowledge, at least for Canada, regional data has not been used to identify cycles and/or permanent components or to identify business fluctuations in general. In fact, the literature has been dedicated to analyze these issues but at an international level using aggregate data. At this respect, see the special issue about business cycles published by *Empirical Economics* in 2002.

On the other side, regional data has been used to analyze predominantly the convergence issue. See, among other references, Coulombe (1999, 2000) and the references mentioned there in. One recent exception is Beine and Coulombe (2002), where regional cycles has been detected with the intention of identifying correlations between them to observe if Canada can be considered as a monetary zone. It is important to point out that in Beine and Coulombe (2002) cycles have been calculated using the Hodrick and Prescott (1997) and Baxter and King (1999) filters which assume symmetry and absence of a theoretical model behind the construction of these fluctuations. It is an important difference with this paper.

As Coulombe (1999) argues, Canada is an interesting case for its density of regional economies which suggests a different set of economies and possibilities of fluctuations and answers to different shocks. In Beine and Coulombe (2002), for example, one interesting result is the fact that Quebec and Ontario appear to be more linked with the economy of the Great Lakes in comparison with the rest of Canada. In the case of the Atlantic regions of Canada, it is relatively poor and dependent of the so-called interprovincial redistribution. On the other hand, the economy of the region named Prairies is based on the extraction of oil and gas and where Manitoba appears as a relatively more diversified economy. Therefore, at the regional level it is possible to find so many differences and possibilities of fluctuations as is found in the aggregate data of European countries, for example.

To summarize, the diversity of regional economies in a large country such as Canada seems to be the first element to support the interest in fluctuations in these economies. The knowledge of particular characteristics of fluctuations can also be important from a regional policy perspective. The presence of negative or positive shocks may have different effects on the aggregate economy than on the regional ones because of their diversity. Consequently, regions should be able to answer differently when a shock arrives. In other words, it is possible to see the total effect observed at the aggregate level (here Canada) as simply a “complicated” combination of the

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3 Here, this region is denoted as the Maritimes.
different regional fluctuations.

Of course there are other methodologies to measure business fluctuations. Such is the case of VAR methods either with or without using long run restrictions to identify transitory and permanent components. See Guay and St-Amant (1997) for further details.

The rest of the paper is organized as follows. Section 2 presents the three methodologies used to extract different measures of the business fluctuations in each Canadian region. A brief presentation of the Hodrick and Prescott (1997) filter and the unobserved components model of Clark (1987) is made. Some details about the model by Hamilton (1989) are also described. Section 3 presents the analysis of the estimates of the permanent and transitory components using all different methodologies. Section 4 offers a tentative chronology for business fluctuations in Canadian regions. Section 5 concludes.

2 Econometric Specifications to Extract Business Fluctuations

In this section, three different methods frequently used to identify business-cycle fluctuations are presented. In order, we present the Hodrick and Prescott (1997) filter (hereafter HP), the unobserved component model (Clark, 1987) and the model of business fluctuations suggested by Hamilton (1989). In the following discussion, we use $y_t$ to represent the logarithm of the real GDP at time $t$ for some particular region or territory.

2.1 The Hodrick and Prescott Filter

The HP filter decomposes a time series $y_t$ into an additive cyclical component $y^c_t$ and a growth component $y^g_t$:

$$y_t = y^c_t + y^g_t. \quad (1)$$

Applying the HP filter involves minimizing the variance of the cyclical component $y^c_t$ subject to a penalty for the variation in the second difference of the growth component $y^g_t$. This is expressed as

$$\{y^g_t\}_{t=0}^{T+1} = \arg \min \sum_{t=1}^{T} [(y_t - y^g_t)^2 + \lambda ((y_{t+1}^g - y^g_t) - (y^g_t - y^g_{t-1}))^2] \quad (2)$$

where $\lambda$, the smoothness parameter, penalizes the variability in the growth component. The larger the value of $\lambda$, the smoother the growth component.
Hence, as $\lambda$ approaches infinity, the growth component corresponds to a linear time trend. For quarterly data, Hodrick and Prescott (1997) propose setting $\lambda$ equal to 1,600. King and Rebelo (1993) show that the HP filter can render stationary any integrated process for up to the fourth order.

Even when the HP filter is widely used to extract business fluctuations, there are important critics of its utilisation. See for example Guay and St-Amant (1996) for explanations about poor results in extracting business-cycle fluctuation frequencies from macroeconomic time series. Other papers related to these issues are Van Norden (1995) and St-Amant and Van Norden (1997), where the problem for estimating the end of the sample point is also raised. However, this methodology is considered for the purpose of comparison, as well as to provide a starting point for this analysis.

### 2.2 The Unobserved Components Model

This model was suggested by Clark (1987). Using same notation as in Kim and Nelson (1999a,b), the unobserved components model can be represented by

\[
y_t = n_t + x_t \tag{3}
\]

\[
n_t = \gamma_{t-1} + n_{t-1} + \nu_t \tag{4}
\]

\[
g_t = g_{t-1} + w_t \tag{5}
\]

\[
x_t = \phi_1 x_{t-1} + \phi_2 x_{t-2} + e_t \tag{6}
\]

where $\nu_t \sim \text{i.i.d. } N(0, \sigma^2)$, $w_t \sim \text{i.i.d. } N(0, \sigma^2_w)$ and $e_t \sim \text{i.i.d. } N(0, \sigma^2_e)$. In above representation, $n_t$ represents a stochastic trend component, and $x_t$ is a stationary cyclical component. The disturbances $\nu_t$, $w_t$ and $e_t$ are independent white noise processes. Notice that the drift term $g_t$ in the stochastic trend component is modeled as a random walk. It is consistent when we are assuming the presence of a decline in productivity growth and a reduction of labor force growth.

The model can be written in state-space form. The observation equation is

\[
y_t = \begin{bmatrix} 1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \tau_t \\ \epsilon_t \\ \epsilon_{t-1} \\ g_t \end{bmatrix} = H\xi_t \tag{7}
\]
while the state equation is

\[
\xi_t = \begin{bmatrix}
0 \\
\pi_{st} \\
0 \\
0
\end{bmatrix} + \begin{bmatrix}
1 & 0 & 0 & 1 \\
0 & \phi_1 & \phi_2 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix} \xi_{t-1} + \begin{bmatrix}
v_t \\
u_t \\
v_t \\
w_t
\end{bmatrix}
\]

\[
= \bar{\mu}_{st} + F\xi_{t-1} + V_t
\]

where \(E(V_tV'_t) = Q_{st} \) and

\[
Q_{st} = \begin{bmatrix}
\sigma_{v_{st}}^2 & 0 & 0 & 1 \\
0 & \sigma_{u_{st}}^2 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & \sigma_w^2
\end{bmatrix}.
\]

### 2.3 The Markov-Switching Model of Business Fluctuations

A way of capturing the idea of GDP as going through high-growth and low-growth stages is to use a Markov-switching model, such as that used by Hamilton (1989). In such models, it is assumed that a time series \(y_t\) can be represented by the general stochastic process:

\[
(\Delta y_t - \mu_{st}) = \sum_{i=1}^{\infty} \phi_i (\Delta y_{t-i} - \mu_{s_{t-i}}) + \epsilon_t
\]

where \(\epsilon_t \sim N(0, \sigma_{\epsilon_t}^2)\). The discrete-valued variable \(s_t\) denotes the state at time \(t\), and the state vector can be assumed to follow a \(k\)-state Markov process. For example in the case where \(\Delta y_t\) represents the growth rate of GDP, each different state represents a different mean rate of growth. The true state is unobservable and has to be inferred from the observations on the series \(\Delta y_t\) up to time \(t\), which are named the filtered probabilities; or on the basis of the complete sample for \(\Delta y_t\), which are named the smoothed probabilities.

Here, a simplified version of this model is used, where the number of possible states is \(k = 2\), as in Hamilton (1989). Furthermore, Hamilton (1989) used an \(AR(4)\) process and the variance was considered constant across states. The transition probabilities are considered to be constant over the sample and follow a first-order Markov process, which means that the transition probabilities are defined by the current \(t-1\) state only. In formal terms, it is represented by

\[
(\Delta y_t - \mu_{st}) = \phi_1 (\Delta y_{t-1} - \mu_{s_{t-1}}) + \phi_2 (\Delta y_{t-2} - \mu_{s_{t-2}}) + \phi_3 (\Delta y_{t-3} - \mu_{s_{t-3}}) + \phi_4 (\Delta y_{t-4} - \mu_{s_{t-4}}) + \epsilon_t
\]

5
where $\epsilon_t \sim N(0, \sigma^2)$ and the roots of $\phi(L) = (1 - \phi_1 L - \ldots - \phi_4 L^4) = 0$ lie outside the unit circle. Furthermore,

$$
\mu_{st} = \mu_0 (1 - s_t) + \mu_1 s_t
$$

(11)

where 0 is denoted to be the low-growth state (recession) and 1 is the high-growth state (expansion). One simple parameterization of the conditional state transition probabilities is to set

$$
\Pr[s_t = 1|s_{t-1} = 1] = p,
$$

(12)

$$
\Pr[s_t = 0|s_{t-1} = 1] = 1 - p,
$$

(13)

$$
\Pr[s_t = 0|s_{t-1} = 0] = q,
$$

(14)

$$
\Pr[s_t = 1|s_{t-1} = 0] = 1 - q.
$$

(15)

Hamilton (1989) used this model structure and applies a filter that iterates over the observed sample, producing as a by-product the sample likelihood. It is a function of both the number of possible patterns of regime shifts in the sample and the number of possible parameter values for the density functions assumed to be generating the data in the $k = 2$ regimes. Numerical optimization yields parameters that maximize the sample likelihood and are easily interpretable in terms of growth rates in high and low states and the conditional probabilities associated with these states. The parameter estimates are consistent and asymptotic standard errors of the parameter estimates can be obtained from a numerical approximation to the Hessian.

Unfortunately, $s_t$ is an unobserved variable. Let $\psi_{t-1}$ be the set of all past information up to time $t - 1$. What we need is to write the density of $y_t$ given past information and to do that we need $s_t$ and $s_{t-1}$, which are unobserved. To solve this problem, instead of considering a joint density of $y_t$ and $s_t$, we consider the joint density of $y_t$, $s_t$ and $s_{t-1}$. Basically, this can be done in two steps as shown in Kim and Nelson (1999a). In the first step, we derive the joint density of $y_t$, $s_t$ and $s_{t-1}$, conditional on $\psi_{t-1}$:

$$
f(y_t, s_t, s_{t-1}|\psi_{t-1}) = f(y_t|s_t, s_{t-1}, \psi_{t-1}) \Pr[s_t, s_{t-1}|\psi_{t-1}],
$$

where

$$
f(y_t|\psi_{t-1}, s_t, s_{t-1}) = (2\pi\sigma_{st}^2)^{-1/2} \exp\left\{-\frac{[\phi(L)(y_t - \mu_{st})]^2}{2\sigma_{st}^2}\right\}.
$$

In the second step, to get $f(y_t|\psi_{t-1})$, we have to integrate $s_t$ and $s_{t-1}$ out of the joint density by summing the joint density over all possible values of
$s_t$ and $s_{t-1}$:

$$f(y_t | \psi_{t-1}) = \sum_{s_t=1}^{k} \sum_{s_{t-1}=1}^{k} f(y_t, s_t, s_{t-1} | \psi_{t-1})$$

$$= \sum_{s_t=1}^{k} \sum_{s_{t-1}=1}^{k} f(y_t | s_t, s_{t-1}, \psi_{t-1}) \Pr[s_t, s_{t-1} | \psi_{t-1}].$$

Then, the marginal density $f(y_t | \psi_{t-1})$ is a weighted average of $k^2$ conditional densities. Finally, the log likelihood is given by:

$$\ln L = \sum_{t=1}^{T} \ln \left\{ \sum_{s_t=1}^{k} \sum_{s_{t-1}=1}^{k} f(y_t | s_t, s_{t-1}, \psi_{t-1}) \Pr[s_t, s_{t-1} | \psi_{t-1}] \right\}.$$ 

To complete the above procedure, we still need to deal with the problem of calculating $\Pr[s_t = j, s_{t-1} = i | \psi_{t-1}]$, that is, the weights. This is known as the filtering procedure which also consists of two steps. In the first step, given $\Pr[s_{t-1} = i | \psi_{t-1}]$, $i = 1, 2, ..., k$, at the beginning of time $t$, we can calculate the weights by

$$\Pr[s_t = j, s_{t-1} = i | \psi_{t-1}] = \Pr[s_t = j | s_{t-1} = i] \Pr[s_{t-1} = i | \psi_{t-1}]$$

where the first term in the last expression are the transition probabilities. In the second step, once $y_t$ is observed at the end of time $t$, we can update the probability terms by

$$\Pr[s_t = j, s_{t-1} = i | \psi_{t-1}] = \Pr[s_t = j | s_{t-1} = i, y_t]$$

$$= \frac{f(s_t = j, s_{t-1} = i, y_t | \psi_{t-1})}{f(y_t | \psi_{t-1})}$$

$$= \frac{f(y_t | s_t = j, s_{t-1} = i, \psi_{t-1}) \Pr[s_t = j, s_{t-1} = i]}{\sum_{s_{t-1}=1}^{M} \sum_{s_t=1}^{M} f(y_t | s_t, s_{t-1}, \psi_{t-1}) \Pr[s_t, s_{t-1} | \psi_{t-1}]}.$$ 

with $\Pr[s_t = j | \psi_t] = \sum_{s_{t-1}=1}^{k} \Pr[s_t = j, s_{t-1} = i | \psi_t]$. Iterating these two steps for $t = 1, 2, ..., T$ allows for the appropriate weighting terms to be used in the log likelihood.

Construction of the log likelihood and filtering steps are known as the Hamilton filter which is a modification of the Kalman filter. In fact, one of the principal outputs from the Hamilton filter are the so-called filtered probabilities, which are inferences about $s_t$ using information until time $t$. However, it is also possible to obtain smoothed probabilities which are
inferences about \( s_t \) using information until time \( T \), that is, total information. This is found using the smooth algorithm proposed by Kim (1994). For further details, see Hamilton (1989, 1993).

3  Empirical Evidence

3.1 Estimates

Figure 1 presents the current output and the estimated potential output using the HP filter. Figure 2 shows deviations from these estimates which can be considered as a measure of the business fluctuations in the economies reviewed. It is clearly observable that regions such as Prairies, Maritimes, Alberta, and Prairies excluding Alberta present more variability in the behavior of their business fluctuations than the other territories. Hence, it is possible to predict that a large number of peaks and troughs will be found in the next section.

Table 1 presents the parameter estimates obtained from the unobserved components model. In the cases of Ontario and Quebec, it is observed that the persistency degree, given by the sum of the autoregressive coefficients, is close to unity. Similar cases are obtained for Canada and the US. In the other cases, we observe that this sum is clearly below unity\(^4\). Then, according to this model, cyclical fluctuations in these territories are less persistent than in the former cases.

Another fact from Table 1 is that in all cases the variance of the cyclical component is statistically different from zero. The variance from the stochastic trend in not statistically significantly different from zero for the cases of British Columbia and Canada. Finally, the variance of the drift term is statistically different from zero for all cases but Ontario and the US. This means that in these cases, the data does not support the existence of a decline in productivity growth and a reduction in the labor force growth.

Figure 3 indicates the behavior of the current output and the estimates of the permanent component of \( y_t \) obtained using the unobserved components model. Unlike the dynamics obtained by the HP filter (Figure 2), now there is clear evidence of periods where certain Canadian regions were operating under their permanent component. The most clear examples are Ontario, Quebec and Canada. A clearer visualization is obtained from Figure 4, which shows the estimated cyclical components.

Table 2 presents the estimates obtained from the Markov-switching \( AR(4) \)

\(^4\) These results are very similar to those obtained by Rodríguez (2003).
model. Most of the estimated parameters are statistically different from zero. The expected duration of an expansion is measured as \((1 - p)^{-1}\) and the expected duration of a recession is given by \((1 - q)^{-1}\). From the estimates in Table 2, it can be seen that the expected duration of an expansion in Ontario and Quebec is 34.5 and 37.0 quarters, respectively. On the other hand, British Columbia and the US show very close expected values (22.6 and 22.7 quarters, respectively), while Alberta and Prairies present the smaller values with 12.6 and 7.5 quarters, respectively. Finally, the region of Maritimes presents the highest value with 66.6 quarters. However, it is also the region which presents the highest expected value for the duration periods of recession (62.5 quarters). All other regions present similar values: 1.4, 3.7, 3.4, 3.3, 4.8, 4.1 and 3.6 quarters for British Columbia, Ontario, Prairies, Quebec, Alberta, Canada and the US, respectively.

As it is known, Nelson and Plosser (1982) and Campbell and Mankiw (1987a, 1987b) were interested in the extent to which recessions represent temporary deviations from potential output with the shortfall largely made up during the subsequent recovery. Earlier approaches of this issue were based on the standard linear representation of a non-stationary time series \(\tilde{y}_t\), that is, \((1 - L)\tilde{y}_t = \mu + \psi(L)u_t\); where \(u_t = \sum_{j=0}^{\infty} \psi_j u_{t-j}\). Hence, the permanent effect on the level of the series of a current innovation \(u_t\) is given by the \(\lim_{j \to \infty} \partial E_t(\tilde{y}_{t+j}) / \partial u_t = \sum_{j=0}^{\infty} \psi_j = \psi(1)\). Some researchers sought a finite representation of \(\psi(L)\) based on Box-Jenkins methods, bivariate models and non-parametric tests (Cochrane, 1988). Precisely, one model used to measure this effect is the unobserved components model, which is linear and does not allow for asymmetries. Using the estimated autoregressive parameters from this model (Table 1), we can observe that the permanent effect on the level of the series is always greater than unity. For Canada and the US, the estimated effect is 1.06 and 1.05, respectively. Similar effects are obtained for Ontario (1.02) and Quebec (1.03). However, for the region of Maritimes this estimate is larger (1.43). Other territories present intermediate values: Prairies (1.16), Alberta (1.15), Prairies excluding Alberta (1.12), and British Columbia (1.19). The results indicate that disturbances affect greatly the region of Maritimes and lesser effects are found in bigger regions as Quebec and Ontario and in the aggregates like Canada and the US.

However, the unobserved components model does not allow for asymmetries and nonlinearities. By contrast, the Markov-switching model is fundamentally nonlinear and provides an alternative perspective on the basic ques-

\footnote{The same results were found when different sets of starting values were tried.}
tion concerning business cycles. Notice that this model can be represented by 

\[(1 - L) \tilde{y}_t = (\mu_0 + \mu_1 s_t) + [\phi(L)]^{-1} \epsilon_t.\]

Then, the two fundamental sources of randomness, \(s_t\) and \(\epsilon_t\), are allowed to have very different impacts for the future path followed by \(\tilde{y}_t\). We could associate \(s_t\) with the business cycle directly and \(\epsilon_t\) with other factors contributing to changes in output. The permanent effect of the non-business cycle component \(\epsilon_t\) is given by the \(\lim_{j \to \infty} \partial E_t(\tilde{y}_{t+j})/\partial \epsilon_t = \phi(1)^{-1}\). On the other hand, if at date \(t\) the economy is in recession \((s_t = 0)\) rather than the growth state \((s_t = 1)\), the consequences for the long-run future level of \((100\text{ times the log of})\) real income is given by

\[
\lim_{j \to \infty} \left[ E_t(\tilde{y}_{t+j} | s_t = 1) - E_t(\tilde{y}_{t+j} | s_t = 1) \right] = \mu_1 (-1 + p + q) / (2 - p - q). \]

See Hamilton (1989) for further details.

Using the estimates in Table 2, the two values mentioned above were calculated. The permanent effect of \(s_t\) is large for Maritimes and Alberta with 16.5% and 4.0%, respectively. The smaller values are obtained for Quebec, Prairies, British Columbia and US with 1.7%, 1.9%, 2.0% and 2.1%, respectively. Ontario with an effect of 2.6% is similar to the value obtained for Canada (2.7%). Estimates of the other source of randomness \((\epsilon_t)\) were also calculated. The larger effects (larger than unity) are found for Quebec (2.2), Canada (1.7), Ontario (1.6), US (1.4) and British Columbia (1.3). Effects lower than unity correspond to Prairies (0.93) and Alberta (0.92). The region of Maritimes presents a value exactly equal to unity. Both kinds of effects allow us to conclude that the randomness corresponding to the business cycle are more important for the region of Maritimes than the randomness coming from \(\epsilon_t\), a feature impossible to determine using the unobserved components model. In the same direction, but to a small proportion, are the cases of Ontario, Albert, Canada, the US and British Columbia. In the other cases the situation is the opposite, that is, the randomness of \(\epsilon_t\) is more important than the randomness from the business cycles.

3.2 A Chronology of the Regional Business Cycles Fluctuations

Figures 2 and 4 can be used to identify observations classified as peaks and troughs, respectively. A peak is defined as the last observation (quarter) before a recession, that is, a negative growth rate. A trough is defined as the last observation (a quarter) of a recession. Here, a recession will be considered as a period with at least two consecutive negative quarters.

Figure 5 presents the probabilities of being in times of recession \((s_t = 0)\) using the Markov-Switching AR(4) model. Notice that the probabilities
for the full sample smoother differ very little from those of the four-lag smoother (filtered probabilities). In constructing a chronology of business cycle fluctuations, the simple rule suggested by Hamilton (1989) is used, which indicates that observations with filtered (or smoothed) probabilities larger than 0.5 are considered as recession times. Again, a recession is considered to be at least two consecutive quarters with this fact.

Dates of peaks and troughs are presented in Tables 3-5 for each territory. Also included in Table 6 is a chronology for Canada and the US obtained from the Economic Cycle Research Institute (2002) which publishes dates of business cycles for a sample of countries. Notice that the same source is used by Bodman and Crosby (2000). The following observations can be extracted from all these tables. The number of peaks and troughs obtained using the HP filter are larger than those obtained using the other two specifications. This shows a greater degree of variability in the estimates obtained from the HP filter. On other hand, the region of Maritimes exhibits the largest number of peaks and troughs compared to any other territory. This result is consistent for all three specifications. Notice that this is consistent with the estimated parameters obtained for this region.

Using the HP filter, recession times seem to be stopped in most regions around 1998. Exceptions occur in British Columbia, Prairies, Canada and the US. A similar result is observed from the dates obtained using the unobserved component model but now, Alberta also presents a recent recession time while Ontario and Prairies appear to have a recession in 1999. Unlike the HP filter, now, Canada and the US appear to have their last recession in 1996:4 and 1997:1, respectively. The territory of Prairies excluding Alberta presents its last recession time in 1996:2, which seems to indicate that only Alberta presents a recent period of recession.

According to the Markov-Switching AR(4) model, only the region of Prairies shows a larger number of peaks and troughs but these are short-lived. Unlike this territory, the region of Maritimes presents a shorter number of periods but they last longer, which is consistent with the estimates obtained from this model. According to this specification, the last recession time in British Columbia occurred in 1986:4; and in Quebec and Ontario, it took place around 1989-1991. The region labeled Prairies presents its last recession time in 1998 which is due particularly to Alberta. On another hand, Canada and the US show their last recessions in 1991.

As it is well known, the National Bureau of Economic Research (NBER)
calculates dates for peaks and troughs for the US and for a selected sample of countries. These dates for Canada and the US are presented in Table 6, which can be used as a reference point for the estimates obtained in the preceding tables. Let us consider the case of the US. Dates estimated using the HP filter have a longer duration than those calculated by the NBER. Moreover, there are around five periods found by the HP filter that are not found by the NBER. See, for example, the case of the two last recession periods in Table 3. In some cases, recession times calculated by the HP filter start before those calculated by the NBER and end later. The same pattern is observed for the dates calculated for Canada.

Now consider the dates estimated by the unobserved components model. As before, the duration of the periods are longer using this model than those dates calculated by the NBER. For example, the last recession time calculated for the model is six year longer than the recession time calculated by the NBER. Moreover, the recession time around the first oil shock is presented in our model, but not in the dates of the NBER. For Canada, we find longer recession times and some periods do not coincide with those suggested by the NBER.

Finally, consider the dates estimated by the Markov-Switching AR(4) model. Almost all dates coincide with those suggested by the method of the NBER. The recession time of 1992:4-1993:1 is not detected by the NBER probably because this period only lasted two quarters. Notice that the peak of 2001:1 calculated by the NBER is outside of our sample. Dates found for Canada are relatively coincident with those reported by the NBER. Comparing with dates estimated by Bodman and Crosby (2000), we consider our estimates as better approximations to those calculated by the NBER. However, as before, there are some periods where the model find recessions periods that do not coincide with the NBER findings.

The final results suggest that recession times calculated by the Markov Switching AR(4) are more closely related to those reported by the NBER. A plausible explanation is the fact that this model allows for asymmetries and nonlinearities compared to the HP filter or the unobserved component model which do not allow for these issues.

In the case of British Columbia, the model reports three recession periods in the 1970s, specially around 1974-1975. Two other recession times are found in the 1980s. In the region of Maritimes, the model suggests two brief recession times around 1966 and 1970 and a longer period starting from 1974. This last period is obviously consistent with the estimates obtained from the model and the estimated expected duration of a recession. Even when this result was robust to many of the sets of different starting values,
it should still be regarded with caution.

Ontario present three clear recession times. The first period occurs in the mid-1970s; the second period appears at the beginning of the 1980s and the last period is reported at the end of the 1980s/beginning of the 1990s.

The region of Prairies, as was mentioned before, presents many recession periods, but they all are of short duration. Quebec presents dates of recession times closely related to those reported for Ontario, with the difference being the period 1976:1-1976:3 where a recession period is found only for Quebec. Finally, Alberta and the Prairies excluding Alberta present around three recession times in ten years.

A set of correlations between the estimates of business fluctuations was also calculated. Estimated correlations using HP filter estimates show some strange results, as for example, the negative association between the cycles of the Prairies and Alberta. It is strange because, by construction, Prairies includes Alberta. It is also possible to find other cases which seem to indicate that the cyclical estimates obtained by HP filter have some drawbacks.

The correlations of cyclical estimates from the unobserved components seem to make more sense. First of all, the strange negative correlations found with HP filter do not exist. Looking at the correlations of the cyclical component in Canada seems to indicate that it is correlated with the cyclical component of the US with a coefficient of correlation of 0.82; and the correlation with Ontario and Quebec is 0.71 and 0.79, respectively. On another hand, Ontario is correlated with Quebec with an estimated coefficient of correlation of 0.85. Other correlations are not larger than 0.60. Similar observations may be obtained observing the smoothed probabilities (to be in recession times) obtained from the Markov-switching model. However, the estimated correlations are not higher.

4 Conclusions

Using three different econometric specifications, the estimates of permanent and business cycles fluctuations have been obtained for Canadian regions. Graphical behavior of the recession times using the unobserved component model (Clark, 1987) and Markov-switching model (Hamilton, 1989) offer a more realistic chronology of the business cycle fluctuation in Canadian regions compared to those obtained from the HP filter. The expected duration of times of recession or expansion show that the region labeled Maritimes is the most probable to have persistent regimes. Expected recession periods are very similar between the regions, as well as when they are compared
to Canada and the US, except for the region of Maritimes, where the expected duration of recession times is higher. For this region it is also possible to confirm that the source of randomness originates from the business fluctuations and not from the innovation to the real output. It confirms that this region is very sensible to these type of randomness. The same results, but to a lesser degree, appear for the province of Alberta.

References


Table 1. Estimates based on the Univariate Unobserved Components Model*

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* Standard errors in parenthesis.

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* Standard errors in parentheses.
### Table 3. Business Cycle Chronologies based on the HP Filter

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### Table 3 (continuation). Business Cycle Chronologies based on the HP Filter

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* Excludes Alberta.
Table 4. Business Cycle Chronologies based on the Unobserved Components Model

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Table 4 (continuation) . Business Cycle Chronologies based on the Unobserved Components Model

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<tr>
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<td>92:3</td>
<td>78:4</td>
<td>79:1</td>
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<tr>
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<td>82:1</td>
<td>82:4</td>
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<td>92:4</td>
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<tr>
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<td>98:4</td>
<td>86:1</td>
<td>88:3</td>
<td>95:1</td>
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</tr>
<tr>
<td>Trough 90:2</td>
<td>92:4</td>
<td>90:3</td>
<td>91:3</td>
<td>01:1</td>
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Table 6. Business Cycle Chronology calculated by the NBER

<table>
<thead>
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<th>United States</th>
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<tr>
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<tr>
<td>90:1</td>
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<td></td>
<td>81:3</td>
</tr>
<tr>
<td></td>
<td>91:3</td>
</tr>
</tbody>
</table>
Figure 1. Permanent Component and Current $y_t$ based on the HP Filter
Figure 2. Transitory Components based on the HP Filter
Figure 3. Permanent Component and Current $y_t$ based on the Unobserved Components Model
Figure 4. Transitory Components based on the Unobserved Components Model
Figure 5. Filtered and Smoothed Probabilities to be in Recession based on the Markov-Switching AR (4) Model